Abstract
Unlike Arrow’s result for process innovations, we show that the gain from a product innovation can be larger to a secure monopolist than to a rivalrous firm that would face competition from independent sellers of the old product. A monopolist incurs profit diversion from its old good but may gain more than a rivalrous firm on the new good by coordinating the prices. In a Hotelling framework, we find simple conditions for the monopolist’s gain to be larger. We also explain why the ranking of incentives differs under vertical product differentiation.


1 Introduction

Does initial market power dilute a firm’s incentive to invest in substitute new technologies in order to protect its existing profit? This question has longstanding interest to policymakers and economists. In a seminal paper, Arrow (1962) showed that a secure monopolist gains less from perfectly patentable process innovations—that lower the marginal cost of an existing homogeneous product—than would a competitive firm facing the same market demand. The logic is simple for a drastic innovation—one that renders the old technology irrelevant. Post innovation profit will then equal the lower-cost monopoly level whatever the initial market structure, so the gain from innovation is lower for an initial monopolist because only it enjoys status quo profit that the innovation cannibalizes or “replaces.” If the innovation is nondrastic (the new monopoly price exceeds the old marginal cost), however, there is an opposing effect: a monopolist that innovates will earn higher profit post innovation than would an innovator that faces competition from the old technology. Nevertheless, Arrow proved that a monopolist’s gain is lower by using a different argument: the value of a process innovation comes solely from reducing the firm’s marginal cost and this cost reduction applies to a smaller output under monopoly than under homogeneous Bertrand competition (see also Tirole 1988).

This output level argument, however, is specific to reductions in marginal cost for a homogeneous product and leaves the open question: Does Arrow’s ranking of innovation incentives across market structures extend to nondrastic product innovations, where the new good is a differentiated substitute for the initial good and does not entirely displace it? Our main finding is that the ranking can be reversed — the incentive to undertake substitute product innovations can be stronger for a monopolist.\(^1\) Compared to a rivalrous firm, a monopolist adding a substitute product will divert profit from its old product, but may gain more from the new product by coordinating the two prices. We show that the coordination effect can dominate.

Specifically, our model compares a firm’s incentive to add a new product \(B\) under three alternative market structures, where the firm’s gain is denoted in parentheses: secure Monopoly \((G^m)\) — a monopolist controls product \(A\) and only that same firm can add \(B\); ex post Duopoly \((G^d)\) — a monopolist controls product \(A\) but only a different firm can add \(B\); and Competition \((G^c)\) — product \(A\) is supplied by homogeneous Bertrand rivals (Arrow’s case), and any firm can add \(B\). One motivation for these comparisons is that policy interventions can alter the market structure and, hence, the innovation incentive, as under the following scenarios.

Merger Scenario: Suppose producers of good \(A\) are competitive, only they have the requisite

\(^1\)We assume the new product is a substitute instead of a complement for the initial good because the view that a monopolist resists innovation to protect its initial profit presumes substitute technologies. (A complementary new technology would enhance rather than devalue the old one.)
assets to add product $B$, and they propose a merger-to-monopoly in $A$. The gain from adding $B$ is $G^m$ if the merger is approved and $G^c$ if the merger is rejected.

**Patent Scenario:** Good $A$ is supplied by a monopolist protected by a patent that also blocks innovation in $B$. If the patent is retained, the innovation incentive is $G^m$; if it is voided, market $A$ becomes competitive, and the innovation incentive changes to $G^c$.

**Regulation Scenario:** Suppose there is a durable franchise monopolist in $A$ and the policy choice is between (i) granting that firm also the monopoly rights over a competing future product $B$, versus (ii) barring that firm from $B$. Under regime (i), the incumbent is unconcerned with preemption in $B$, so its gain from adding $B$ is given by $G^m$; under regime (ii), only an entrant can innovate in $B$, and its gain is given by its profit under ex post duopoly, $G^d$. Option (ii) may be motivated by a (correct) concern that the incumbent would price the new product higher than would an entrant.

We represent product differentiation as horizontal, following the classic framework of Hotelling (1929; see also Tirole 1988), adapted to allow asymmetries: products $A$ and $B$ can differ in their ‘qualities’ — their value to consumers gross of transport costs — or (equivalently for modeling purposes) in their marginal costs. We show that $G^m > G^d$ if and only if the new product has higher quality than the old, and that $G^m > G^c$ always holds. Thus, the incentive for product innovation can be greater under secure monopoly than under market structures that admit product market rivalry. In some such cases, overall welfare also is higher under monopoly. Of course, these findings should not be construed as advocating monopoly since our analysis abstracts from various potential inefficiencies of monopoly, such as internal slack (Raith 2003; Schmidt 1997). The results merely caution against sweeping claims that monopoly invariably undermines product innovation.

Our model purposely omits several important factors discussed in the literature that can influence relative innovation incentives under alternative market structures, such as imperfect patent protection, the stochastic nature of R&D outcomes, and scope for follow-on innovations. (See, for example, Vickers 2010, Gilbert 2006, Reinganum 1989, and Tirole 1988.) We retain Arrow’s environment of perfect patent protection and a deterministic, one-shot innovation in order to isolate the difference between process and product innovations in a common setting.

Gilbert and Newbery’s (1982) well known paper essentially considers a similar deterministic bidding setting but with an important difference: the initial monopolist is not secure and foresees that if it does not adopt the innovation an entrant will. The added incentive to foil entry leads the monopolist to outbid an entrant. Spulber (2012) relaxes Gilbert and Newbery’s assumption that industry profit is always maximized with a single firm and analyzes the types of innovations for which entry will occur. Unlike their papers, we address a secure monopolist (or a monopolist
unaware of the entry threat) that lacks this strategic incentive to innovate.

To our knowledge, the only authors who have compared the product innovation incentives of a secure monopolist and a rivalrous firm in a setting similar to ours are Greenstein and Ramey (1998) and Gilbert (2006). Gilbert shows that Arrow’s scale effect favoring a Bertrand competitor’s incentive to innovate relative to a monopolist’s can be reversed if the initial competition is in differentiated rather than homogeneous products. In Gilbert’s example the innovator discards the old product, which is dominated by the new one, akin to a process innovation.\(^2\) Our analysis considers cases where the products are imperfect substitutes and a monopolist would sell both. The paper closest to ours is Greenstein and Ramey (1998). Their framework is the same as ours, but they analyze vertical product differentiation (Shaked and Sutton 1983, Tirole 1988). Importantly, the ranking of incentives differs. They find that \(G^m = G^c < G^d\),\(^3\) suggesting that product innovation incentives are lower under protected monopoly than under product market rivalry. By contrast, we find that \(G^m > G^c\) and that \(G^m > G^d\) also can hold. The price coordination advantage of a two-product monopolist over an innovator that would compete against the old product is stronger under Hotelling product differentiation than under vertical product differentiation for reasons we shall explain.

The remainder of the paper is organized as follows. Section 2 describes our model, and characterizes equilibrium prices, outputs, and profits under alternative market structures in this simple asymmetric Hotelling setting. Section 3 presents our main results. We derive the precise conditions under which the incentive to add a new product is higher or lower for a secure monopolist than for a firm that will face product market rivalry, and we also report findings on welfare comparisons. In Section 4, we further discuss our results in the context of the relevant literature. Concluding remarks are offered in Section 5.

2 The Model

The market has an initial product, \(A\). An innovation will bring a new product, \(B\), to the market. The innovator obtains exclusive rights over \(B\). Products \(A\) and product \(B\) (if it is innovated) are located at the two end points of an Hotelling line, \(x = 0\) and \(x = 1\), respectively, with their

\(^2\) In Gilbert’s model, consumers are located on a Hotelling line, have unit demands, and the market is covered under monopoly or competition, generating fixed total sales normalized to 1. All consumers place an equal premium \(X\) on the new product over the old (so the innovation is like a cost reduction). If a monopolist located at 0 innovates, it raises price by \(X\) to the whole market so its gain is \(X\). Now suppose that the innovating firm located at 0 competes with a rival at the other end. If the products were homogeneous (zero transport costs), the innovator would capture the market with a trivial price cut and its gain would be \(X\), as under monopoly; with differentiation, its gain is lower because to capture the market would require an increase in margin smaller than \(X\).

\(^3\) The equality is shown in their Proposition 2(a), and the inequality in their Proposition 4(a).
prices being $p_A$ and $p_B$, and their constant unit costs $c_A$ and $c_B$.

A unit mass of consumers, each having a unit demand, are uniformly distributed on the Hotelling line. When purchasing a unit of product $A$ or product $B$, a consumer at location $x \in [0, 1]$ receives net surplus $u_A = v_A - tx - p_A$ and $u_B = v_B - t(1 - x) - p_B$, respectively. If $v_A = v_B$, the setting is the standard Hotelling model with pure horizontal product differentiation. Our formulation allows also quality differentiation: if $v_B > v_A$, then product $B$’s quality is higher, in the sense that an equidistant consumer values $B$ more than $A$, and conversely if $v_B < v_A$.

Since $v_A$ is common to all consumers of good $A$, and similarly with $v_B$, what matters for profit functions and equilibrium values are the differences between quality and unit cost of the two products, $v_A - c_A$ and $v_B - c_B$. To simplify notation, we set $c_A = c_B = 0$, and analyze differences in quality.\(^4\)

We maintain the following assumption:

**Assumption 1.** 1.1) $v_A \geq 2t$, 1.2) $v_A + v_B > 3t$, and 1.3) $|v_B - v_A| < 2t$.

Assumption 1.1) implies that a monopolist over just good $A$ would cover the market and would set $p_A = v_A - t$; 1.2) implies that the market would also be covered under duopoly; and 1.3) implies that when both products are present, each will have a positive output under either a two-good monopoly or under duopoly.

Depending on whether the old product is initially monopolized or produced competitively, the equilibrium prices and profits in the market, with or without product $B$, can be characterized as follows:

**Old Product Is Initially Monopolized**

With only product $A$, the optimal monopoly price and output are, respectively:

$$p_A^m = v_A - t, \quad q_A^m = 1,$$

and the monopoly profit is

$$\pi_A^m = v_A - t.$$  \hspace{1cm} (1)

If that same firm adds product $B$, it becomes a monopolist over both goods. For prices $p_A$ and $p_B$, the consumer who is indifferent between products $A$ and $B$ is located at $x_i$, given by

\[^4\text{All our ensuing results extend to positive marginal costs by interpreting the valuations } v_A, v_B \text{ and the prices } p_A, p_B \text{ as net of the respective costs. Thus, our results hold if the asymmetries are driven by cost, by quality, or some combination.}\]
\[ v_A - p_A - t x_i = v_B - p_B - t (1 - x_i), \] or:

\[ x_i = \frac{t + v_A - v_B + p_B - p_A}{2t}. \] (3)

Thus, the demand functions are \( q_A = x_i(p_A, p_B) \), and \( q_B = 1 - x_i(p_A, p_B) \). The multiproduct monopolist’s profit is

\[ \pi^{mm} = p_A x_i + p_B (1 - x_i) . \]

Profit-maximization for the monopolist implies that the indifferent consumer will receive zero surplus:

\[ v_A - p_A - t x_i = 0. \]

Substituting for \( x_i \) from (3) shows the relation between the highest prices that maintain market coverage:

\[ p_B = v_A - p_A - t + v_B. \] (4)

We can then express \( \pi^{mm} \) as a function of only \( p_A \). The first-order condition \( d\pi^{mm}/dp_A = 0 \) here is sufficient for profit maximization and implies the following equilibrium prices and outputs for the two-product monopolist:

\[ p_A^{mm} = \frac{3v_A + v_B}{4} - \frac{t}{2}; \quad q_A^{mm} = \frac{1}{2} + \frac{v_A - v_B}{4t}; \]
\[ p_B^{mm} = \frac{3v_B + v_A}{4} - \frac{t}{2}; \quad q_B^{mm} = \frac{1}{2} + \frac{v_B - v_A}{4t}. \] (5)

Compared to single-product monopoly, observe the following. When \( v_A = v_B \), \( q_A^{mm} = \frac{1}{2} \) and \( p_A^{mm} = p_A^m + \frac{t}{2} \). Adding \( B \) lets the firm cover the market at \( p = v - \frac{t}{2} \), thus raising price by \( \frac{t}{2} \) since the marginal consumer is now located at \( x = \frac{1}{2} \) instead of \( x = 1 \). If \( v_B \) increases, the monopolist diverts sales to \( B \) from \( A \) (\( \Delta q_B = -\Delta q_A > 0 \)), while raising both prices. To see why the price of \( A \) must rise, suppose that \( p_A \) were held constant and \( p_B \) were raised equally with \( v_B \) to maintain zero surplus for the original indifferent consumer. Quantities would remain unchanged, but this allocation is no longer optimal: since only the margin on \( B \) has risen, the monopolist gains by shifting sales from \( A \) to \( B \). To do so while holding the new indifferent consumer at zero surplus, it scales back the price increase on \( B \) and raises the price of \( A \) equally. (Thus, in equilibrium the quality-adjusted price of \( B \) falls by \( \Delta v_B/4 \) and \( p_A \) rises by \( \Delta v_B/4 \).)

5Since all consumers purchase when only product \( A \) is offered by the monopolist, it must be true that all consumers purchase when the monopolist offers both products.
The monopolist’s profits from products $A$ and $B$ are:

\[
\pi_{A}^{mm} = \frac{1}{4} (3v_A - v_B - 2t) \left( \frac{v_B - v_A + 2t}{4t} \right),
\]
\[
\pi_{B}^{mm} = \frac{1}{4} (3v_B + v_A - 2t) \left( \frac{v_A - v_B + 2t}{4t} \right).
\]  

(6)

Next, again with an initial monopolist in $A$, if a different firm adds $B$, the market becomes a duopoly. The profit functions for the duopolists are $\pi_{A} = p_A x_i(p_A, p_B)$ and $\pi_{B} = p_B(1 - x_i(p_A, p_B))$, where $x_i$ is given in (3) except that the relevant prices now are the duopoly prices $p_{A}^d$, $p_{B}^d$. From the first-order conditions $\partial \pi_{A}/\partial p_A = 0$ and $\partial \pi_{B}/\partial p_B = 0$, we obtain the duopoly equilibrium prices and outputs as

\[
p_{A}^d = t + \frac{v_A - v_B}{3}, \quad p_{B}^d = t + \frac{v_B - v_A}{3};
\]
\[
q_{A}^d = \frac{1}{2} + \frac{v_A - v_B}{6t}, \quad q_{B}^d = \frac{1}{2} + \frac{v_B - v_A}{6t}.
\]  

(7)

The profits for the two firms are:

\[
\pi_{A}^d = \frac{(v_A - v_B + 3t)^2}{18t}, \quad \pi_{B}^d = \frac{(v_B - v_A + 3t)^2}{18t}.
\]  

(9)

It can be easily verified that all consumers will have positive surplus and will thus indeed purchase given $v_A + v_B > 3t$ (Assumption 1.2).

Observe that if $v_B$ increases, then $p_A$ falls under duopoly but rises under the two-product monopoly (compare (7) and (5)). Moreover, as $v_B$ increases, the gap between $p_B$ and $p_A$ rises more slowly under monopoly than under duopoly: $\partial (p_{A}^{mm} - p_{B}^{mm})/\partial v_B = 1/2 < 2/3 = \partial (p_{B}^{d} - p_{A}^{d})/\partial v_B$. Thus, $q_B$ increases faster at the expense of $q_A$ under monopoly than under duopoly since, using (3), $\partial q_B/\partial v_B = [1 - \partial (p_B - p_A)/\partial v_B]/2t$.\(^6\) We summarize these observations in the following Remark.

**Remark 1** Given the quality of the old good, $v_A$, an increase in the quality of the new good, $v_B$, will cause: (i) the price of $A$ to fall under duopoly but rise under monopoly; and (ii) the market share of $B$ to rise faster under monopoly.

Remark 1 illustrates sharply the coordination advantage in pricing of a two-product monopolist

\(^6\)This discussion further implies that when $B$ is the better product, its market share will be larger under monopoly than under duopoly (and smaller when $B$ is weaker): using (3), $q_B - q_A = (v_B - v_A) - (p_B - p_A)$ and $(p_B - p_A)$ has the same sign as $(v_B - v_A)$ but is smaller in absolute value under monopoly. This market share discrepancy is relevant for our later discussion of drastic innovations.
over an entrant that sells the new good $B$ and competes against a different seller of $A$.

**Old Product Is Initially Perfectly Competitive**

Instead of monopoly in $A$, now suppose there are $n \geq 3$ symmetric and homogeneous Bertrand firms, each earning $\pi_A^c = 0$. This corresponds to Arrow’s (1962) competition case, except that here the innovation brings a new product instead of a cost reduction on the old product. If any firm adds product $B$, the market structure will entail monopoly in $B$ and competitive pricing in $A$, $p_A = c_A$. For brevity, we call this hybrid regime “Competition.”

The innovator’s profit, denoted $\pi_B^c$, will be lower than under ex post duopoly ($\pi_B^c < \pi_B^d$) because the price of the substitute good $A$ will be higher when $A$ is sold by a single firm (the initial monopolist) than when $A$ is competitive.

Specifically, when the innovator of $B$ faces perfect competition in good $A$, the equilibrium price and quantity of $B$ are

$$p_B^c = \frac{v_B - v_A + t}{2}, \quad q_B^c = \frac{v_B - v_A + t}{4t};$$

and the equilibrium profit from $B$ is

$$\pi_B^c = \begin{cases} 0 & \text{if } v_B - v_A \leq -t, \\ \frac{(v_B - v_A + t)^2}{8t} & \text{if } v_B - v_A > -t. \end{cases}$$

### 3 Main Results

Here we present our main results, first ranking innovation incentives across market structures and then reporting some welfare comparisons.

#### 3.1 Ranking the Innovation Incentives

If the seller of $A$ is an initial monopolist and innovates by adding product $B$, it becomes the two-product monopolist. Its gain from adding product $B$ is

$$G^m = \pi_A^{mm} + \pi_B^{mm} - \pi_A^m.$$
If an entrant adds product $B$, the market becomes a duopoly. The entrant’s gain is

$$G^d = \pi^d_B.$$  \hfill (13)

Thus, the difference in incentives is

$$G^m - G^d = \left(\pi^{mm}_B - \pi^d_B\right) - \left(\pi^m_A - \pi^{mm}_A\right).$$ \hfill (14)

Here, $(\pi^m_A - \pi^{mm}_A) > 0$ is the diversion effect on product $A$: only the initial monopolist internalizes the fact that its profit from good $A$ falls when it adds product $B$. The term $(\pi^{mm}_B - \pi^d_B)$ is the coordination effect on product $B$: profit from the new product will generally differ between a two-product monopolist and an independent innovator, because only the monopolist can coordinate the prices of the two goods to maximize its overall profit.

Intriguingly, the coordination effect may be negative. If the new product is significantly weaker than the old (here, if $v_B$ is sufficiently lower than $v_A$), the coordination of prices by the two-product monopolist may actually lead to a lower profit from product $B$ compared to that under ex post duopoly. For instance, if $t = 1$ and $v_A = 2.5$, then for $v_B \in (0.5, 1.1)$, $\pi^{mm}_B < \pi^d_B$. Notice that a positive coordination effect is a necessary but not sufficient condition for the monopolist to have a higher innovation incentive than the entrant, because only the monopolist experiences a profit diversion from the old product.

Using the relevant profit expressions from (2), (6), (9) and performing some algebra yields:

$$\left(\pi^{mm}_B - \pi^d_B\right) - \left(\pi^m_A - \pi^{mm}_A\right) = \frac{[12t + 5(v_B - v_A)](v_B - v_A)}{72t}.$$

Assumption 1.3), $|v_B - v_A| < 2t$, implies $[12t + 5(v_B - v_A)] > 0$. Thus,

$$G^m - G^d \gtrless 0 \text{ if } v_B \gtrless v_A.$$ \hfill (15)

We have therefore established the following result:

**Proposition 1** Under Assumption 1, if the new product $B$ has higher quality ($v_B > v_A$), then the incentive to add $B$ is greater under Monopoly than under the Duopoly regime ($G^m > G^d$); conversely, if the new product $B$ has lower quality, the incentive to add $B$ is lower under Monopoly.

The above result can be understood by starting with symmetry, $v_A = v_B$. If a monopolist over $A$ adds product $B$, it sets equal prices and continues serving the whole market, but raises

\footnote{Comparing $G^m$ and $G^d$ is relevant, for example, for the Regulation Scenario discussed in the Introduction.}
price by \( t/2 \), so its gain is \( G^m = t/2 \). If, instead, an entrant adds product \( B \), its margin in the duopoly competition with the supplier of \( A \) is \( t \), but it only captures \( 1/2 \) the market. Its gain is \( G^d = \pi_B^d = t/2 \), the same as for a monopolist.

Next, starting at \( v_A = v_B \), consider increasing \( v_B \) by a small amount \( \delta \). From (5), a two-good monopolist would raise \( p_B \) by \( \delta 3/4 \) and \( p_A \) by \( \delta 4/ \). Since each good initially has half the sales and diversion from \( A \) to \( B \) is neutral when starting with equal margins, the first-order change in profit is just the average price increase, \( \delta/2 \). Under duopoly, using (7) and (8), \( p_B \) would only rise by \( \delta/3 \) and only to \( 1/2 \) the market, for a gain of \( \delta/6 \); in addition, sales of \( B \) expand by \( \delta/6t \) and the initial margin is \( t \), so the first-order increase in profit is \( \delta/3 \). Thus, an increase in the value of the new good \( B \) raises profit by more when \( B \) is added by the monopolist than by the entrant, showing that \( G_m > G_d \) if and only if \( v_B > v_A \) in the neighborhood of symmetric products. Straightforward algebra shows that, in fact, \( \partial (\pi_A + \pi_B) / \partial v_B > \partial \pi_B^d / \partial v_B \) over the entire relevant range, hence \( G^m = G^d \) at \( v_B = v_A \) implies \( G^m > G^d \) if and only if \( v_B > v_A \). A similar argument implies \( G^m < G^d \) if and only if \( v_B < v_A \).

Now suppose instead that product \( A \) is initially perfectly competitive. The gain to the innovator is then \( G_c = \pi_B^c \). From (2), (6), and (11), the difference in incentives under the monopoly and competition regimes is

\[
G^m - G^c = \begin{cases} 
G^m - 0 > 0 & \text{if } v_B - v_A \leq -t \\
\pi_A^m + \pi_B^m - \pi_A^c - \pi_B^c = \frac{2v_B - 2v_A + 3t}{8} > 0 & \text{if } v_B - v_A > -t 
\end{cases} 
\]

We have thus established the next result:

**Proposition 2** Under Assumption 1, the incentive to add the new product \( B \) is always greater under Monopoly than under Competition.

The incentive to add the new product \( B \) under competition is generally lower than that under ex post duopoly \( (G^c < G^d) \), due to innovator’s lower profit under competition \( (\pi_B^c < \pi_B^d) \), which in turn is due to the lower equilibrium price for product \( A \) under competition \( (p_A^c < p_A^d) \). In our asymmetric Hotelling setting, \( G^d < G^m \) only when \( v_B > v_A \), whereas \( G^c < G^m \) holds for all parameter values of the model.

The preceding analysis has considered situations where innovation is nondrastic — the old product continues to influence the innovator’s equilibrium profit. It is worth noting that, unlike a process innovation, a given product innovation can be drastic under one market structure but

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\( ^9 \) Comparing \( G^c \) and \( G^m \) is relevant, for example, in the Merger scenario or Patent scenario discussed in the Introduction.
not another.\textsuperscript{10} Specifically, in our setting (and in Greenstein and Ramey 1988, Proposition 1) product innovations are drastic for a broader range of parameter values under monopoly than when the innovator faces rivalry from the old good.\textsuperscript{11} Intuitively, a separate firm would price the old product lower than would a joint monopolist because only a monopolist internalizes the profit diversion imposed on the new product. Thus, the old product maintains a constraining influence when available to a rival firm even in some cases where a joint monopolist would shut it down.

There is a common logic between the drastic innovation discussion above and an earlier finding: if the product innovation is nondrastic and weak ($v_B$ is sufficiently smaller than $v_A$), a two-product monopolist earns lower profit on the weaker new product $B$ than would an entrant, $\pi_B^{mm} < \pi_B^d$. (See the example before Proposition 1.) The logic is that a two-product monopolist will price the weaker product so as to divert sales to the stronger product more aggressively than would a competitor selling only the weaker product. If the product innovation is drastic, the weaker product is the old one ($v_A$ is sufficiently smaller than $v_B$), and a monopolist prices it out of the market in some cases where a competitor would maintain it.

### 3.2 Welfare Comparisons

#### Total Welfare

We have shown that when $v_B > v_A$, the incentive to introduce product $B$ is greater for a secure monopolist than for a would-be duopolist. Suppose now that adding product $B$ entails a fixed cost $k$. For certain values of $k$, product $B$ will be introduced under monopoly, but not otherwise. Can overall welfare, therefore, be higher under monopoly?

The answer is not immediate, since part of the monopolist’s gain from adding the new product comes at the expense of consumers, as we show later. However, the following argument demonstrates that total welfare can be higher under monopoly.

Let $S$ denote consumer surplus, $W$ denote total welfare, and $\Delta$ denote the change from pre-innovation to post-innovation. If product $B$ is not introduced, the market is served fully with

\begin{footnotesize}
\textsuperscript{10}In Arrow’s analysis, a process innovation is drastic if the new monopoly price is below the old marginal cost: the innovator then prices as an unconstrained lower-cost monopolist whatever the pre-innovation market structure.

\textsuperscript{11}Under monopoly, $q_A^{mm} = 0$ if $v_B > v_A + 2t$, from (5); whereas under duopoly, $q_A^d > 0$ as long as $v_B < v_A + 3t$, from (8) (the same condition maintains $q_A > 0$, i.e., under the competition instead of duopoly regime). Moreover, even when the old product has no sales, under duopoly or competition it still constrains the price of the new good below the level that a joint monopolist would set. A joint monopolist that shuts down good $A$ and covers the market with $B$ will set $p_B^{mm} = v_B - t$, to yield zero surplus for the consumer located furthest from $B$; but when $A$ is available to a competitor, to cover the market the seller of $B$ must set $p_B$ below $v_B - t$, by an amount $v_A$, the surplus available to the consumer located furthest from $B$ if it bought $A$ at cost (which we normalized to zero). Thus, in our Hotelling model a product innovation can be drastic under monopoly ($\pi_B^d = \pi_B^{mm}$) but is never drastic under a rivalrous market structure ($\pi_B^d < \pi_B^{mm}$).
\end{footnotesize}
product A. If product B is introduced by the monopolist, the change in total welfare is

$$\Delta W = G^m - k + \Delta S,$$  \hspace{1cm} (17)

since $G^m$ is the monopolist’s gain before subtracting the fixed cost. In order for the product to be introduced under monopoly but not under duopoly, $k$ must lie in the range

$$G^d = \pi^d_B \leq k < \pi^{mm}_A + \pi^{mm}_B - \pi^m_A = G^m.$$

(18)

Suppose $\pi^d_B = k$. Then (17) becomes

$$\Delta W = G^m - G^d + \Delta S.$$  \hspace{1cm} (19)

Recall that $v_B > v_A$ if and only if $G^m - G^d > 0$. Furthermore, from the expression preceding (15),

$$G^m - G^d = \frac{[12t + 5(v_B - v_A)](v_B - v_A)}{72t},$$

which increases in $(v_B - v_A)$. As $(v_B - v_A)$ increases towards its limit of $2t$ (Assumption 1.3), good B’s share of the market increases towards 1 and A’s share falls towards 0. Thus, consumer surplus under two-good monopoly becomes arbitrarily close to the level under single-good monopoly: Product B instead of A now serves (almost) all the market, and the monopolist fully appropriates the rise in product quality $v_B - v_A$ through an increase in price. The losing consumers are those who still buy A, but their mass can be made arbitrarily small. Therefore, as $v_B - v_A$ increases towards $2t$, $\Delta S$ remains negative but becomes arbitrarily small, while $G^m - G^d$ is positive and increases, showing that in (19) $\Delta W > 0$.

We therefore have:

**Proposition 3** Given the parameter $t$, there exist parameter values $k, v_A, v_B$, with $v_B > v_A$, such that product B is added under monopoly but not under duopoly and total welfare is higher under monopoly.

**Consumer Welfare**

With the Hotelling setting we have adopted, the introduction of the new product by the monopolist necessarily harms consumers. To see this, observe that with only product A, $p^m_A = v_A - t$ so the average consumer, located at $x = 1/2$, earns surplus of $t/2$. With both products offered, the indifferent consumer is located at $\hat{x} = \frac{v_A - v_B + p_B - p_A + t}{2t} \in (0, 1)$. The monopoly prices
(\(p_{A}^{mm}, p_{B}^{mm}\)) are set to leave this consumer zero surplus:
\[
v_{A} - p_{A}^{mm} - t\hat{x} = 0 = v_{B} - p_{B}^{mm} - t(1 - \hat{x}).
\]

Consumers located at \(x < \hat{x}\) continue buying good \(A\). The average such consumer is located at \(\hat{x}/2\) and earns surplus \(t\hat{x}/2\), less than the surplus \(t/2\) earned by the average consumer when only good \(A\) was supplied. Consumers located at \(x > \hat{x}\) switch to buying product \(B\). The average such consumer is located at \((\hat{x} + 1)/2\) and therefore earns surplus
\[
v_{B} - p_{B}^{mm} - t[1 - (1 + \hat{x})/2] = t(1 - \hat{x})/2.
\]

When only product \(A\) was available, that same consumer earned equal surplus:
\[
v_{A} - p_{A}^{m} - t(\hat{x} + 1)/2 = t - t(\hat{x} + 1)/2 = t(1 - \hat{x})/2.
\]

Thus, consumers that switched from product \(A\) to \(B\) in total earn the same surplus, while those who continue buying \(A\) have lost.

In Chen and Schwartz (2010, Appendix A), however, we present an extension of the Hotelling model where the monopolist’s product innovation can benefit consumers. Thus, consumer welfare and overall welfare can both be higher under monopoly than under more rivalrous regimes when the incentive to innovate is higher under monopoly.

### 4 Discussion

We now compare our analysis and results in more depth to some literature noted earlier. Instead of a secure monopolist, Gilbert and Newbery (1982) consider a threatened monopolist who foresees that if it does not acquire the innovation by a known date, entry is sure to occur. With a nondrastic product innovation, if entry occurs, the incumbent will earn duopoly profit \(\pi_{A}^{d} < \pi_{A}^{m}\) and the entrant will earn \(\pi_{B}^{d}\), whereas if the incumbent preempts it will earn \(\pi_{A}^{mm} + \pi_{B}^{mm}\). Gilbert and Newbery compare the incentives of a preempting monopolist \((G^{mp})\) and an entrant \((G^{d} = \pi_{B}^{d})\) and express the difference as
\[
G^{mp} - G^{d} = (\pi_{A}^{mm} + \pi_{B}^{mm} - \pi_{A}^{d}) - \pi_{B}^{d} = (\pi_{A}^{mm} + \pi_{B}^{mm}) - (\pi_{A}^{d} + \pi_{B}^{d}),
\]
the difference between industry profits under a two-product monopoly (which arises if the incumbent preempts) versus under duopoly. Assuming no significant diseconomies of scope, monopoly
— by avoiding revenue dissipation — yields higher industry profit.\textsuperscript{12} Gilbert and Newbery assume this condition, sometimes termed the “efficiency effect” where efficiency refers only to profits. In their setting, the incumbent monopolist therefore outbids the entrant for a nondrastic innovation that is certain to occur by a known date (so “replacement” would occur regardless).\textsuperscript{13}

Our Hotelling model satisfies Gilbert and Newbery’s assumption that industry profit is higher under a two-product monopoly than under duopoly (compare expressions (6) and (9)). But we consider the incentives of a secure rather than a preempting monopolist. The difference can be decomposed as follows:

\[
G^{mp} = \pi_{A}^{mm} + \pi_{B}^{mm} - \pi_{A}^{d} = [\pi_{A}^{mm} + \pi_{B}^{mm} - \pi_{A}^{m}] + (\pi_{A}^{m} - \pi_{A}^{d})
\]

\[
= G^{m} + (\pi_{A}^{m} - \pi_{A}^{d})
\]

(21)

where \(G^{m}\) is given in (12). A preemptive monopolist’s incentive therefore exceeds that of a secure monopolist by \((\pi_{A}^{m} - \pi_{A}^{d})\): the drop in profit from the initial monopoly level to duopoly that is prevented by foiling entry.\textsuperscript{14} Since \(G^{mp} > G^{m}\), Gilbert and Newbery’s ranking \(G^{mp} > G^{d}\), based on the assumed “efficiency effect,” does not answer our question whether \(G^{m} > G^{d}\) — whether the incentive of a secure monopolist to add a product can exceed an entrant’s incentive (under an alternative market structure).

Spulber (2012) documents that entry by innovators ("entrepreneurship") is common even into established industries, that is, new technologies often are not sold exclusively to incumbents. This suggests that an incumbent faces obstacles to exploiting new technologies, such as diseconomies of scope or various frictions in technology transfer. Invoking such factors, Spulber relaxes the assumption that industry profit is always higher under monopoly than with entry (the “efficiency

\textsuperscript{12}If the incumbent monopolist simply mimicked the duopoly equilibrium it would attain the same industry profit, but generally can do better by coordinating the outputs or prices of the two products. On the strategic limitations of this mimicking argument see Lewis (1983), Judd (1985) and Malueg and Schwartz (1991). Chen (2000) modifies Gilbert and Newbery’s assumptions by letting the entrant produce also the old product, and shows that its incentive to add the new product will exceed the incumbent’s incentive if the products are strategic substitutes and conversely for strategic complements.

\textsuperscript{13}Reinganum (1983, 1989) shows that if the innovation’s arrival date is not deterministic but a stochastic function of firms’ R&D spending, as in patent races, the monopolist will be more willing than an entrant to reduce its R&D and accept a delay, given its status quo profit. This probabilistic replacement effect leads an incumbent monopolist to bid less than an entrant when the innovation is drastic or, by continuity, close enough to drastic. See also Ghemawat (1997, chapter 5). Chen and Schwartz (2010, Section 2) discuss additional contributions to the preemption literature by Salant (1984), Gilbert and Newbery (1984), Galini and Winter (1985), Katz and Shapiro (1987), and Gans and Stern (2000).

\textsuperscript{14}This decomposition is also implicit in Spulber (2012), Proposition 2. When the innovator has all the bargaining power (\(\alpha = 1\), its incentive (Spulber’s \(V^{I}\)) equals that of the preempting monopolist (our \(G^{mp}\)), which exceeds the incentive of a non-threatened monopolist (Spulber’s \(V^{m}\), our \(G^{m}\)) by the difference between the incumbent’s initial monopoly profit and its duopoly profit if entry occurs.
effect”). He determines the profit ranking endogenously by analyzing the tradeoffs: entry creates price competition, but can allow cost reductions and product variety that could not be attained just by selling the innovation to the incumbent. His paper focuses on identifying the types of innovation (explained shortly) for which entry will occur in equilibrium even though an incumbent monopolist is aware of the entry threat.

Spulber considers an incumbent monopolist with marginal cost \( c_1 \). An inventor develops an innovation that has both a product dimension and a process dimension: the new good is differentiated from the old and produced at lower cost \( c_2 \); in some cases, \( c_2 \) may also be applied to the old good. Product differentiation is represented as horizontal via a symmetric quadratic utility function for a representative consumer, which yields linear demands with substitution parameter \( b \); the products are unrelated if \( b = 0 \) and become closer substitutes as \( b \) approaches 1.

In Spulber’s main model (sections 3-4), an inventor chooses between selling the innovation to the incumbent or entering the market itself. Spulber considers four alternative scenarios to capture potential limitations on the incumbent’s ability to use the new technology: the new technology is fully transferable; it is not transferable; only the new cost \( c_2 \) is transferable (not the new product design); or only the new product design is transferable. Full transferability yields Gilbert and Newbery’s preemption result (Spulber’s Proposition 1). To illustrate Spulber’s novel findings, consider the other polar case of non-transferability. His Proposition 3 states that entry will occur if and only if (i) the new cost \( c_2 \) is below some threshold (that depends on \( c_1 \) and \( b \)); or (ii) the substitution parameter \( b \) either is above a high threshold or below a low threshold (both of which depend on \( c_1 \) and \( c_2 \)). Part (i) suggests that entry is more likely for larger cost reductions: the gain from exploiting the lower cost, which can only be done through entry, then outweighs the revenue loss from competition that entry induces. In Part (ii), high substitution allows the innovator with its cost advantage (\( c_2 < c_1 \)) to enter and profitably displace the incumbent. Low substitution allows the entrant to profit from product differentiation (which again, cannot be done by selling the innovation to the incumbent) without unleashing intense price competition. These results establish an interesting non-monotonic pattern: if an innovation is not transferable to the incumbent, entry is more likely to occur when the new product is either a close substitute for the old or highly differentiated from it. For intermediate substitutability, the innovation is sold to the incumbent and monopoly persists.

Unlike Spulber, our analysis does not address whether entry will occur into an initially monopolistic industry but instead compares the innovation incentives of a secure monopolist and a rivalrous firm under an alternative market structure. Greenstein and Ramey (1988) compare the same market structures as we do, but instead of Hotelling they assume vertical product differentiation (Shaked and Sutton, 1983; Tirole, 1988). They find equal incentives to undertake a
nondrastic product innovation under the secure monopoly and competition regimes, $G^m = G^c$ (Proposition 2(a)), unlike $G^m > G^c$ in our Hotelling model.\(^{15}\)

The key difference between vertical and Hotelling differentiation is the identity of the marginal consumers and what this implies for pricing by a two-product monopolist. Under vertical differentiation, the consumers attracted to the new, higher-quality good are the relatively high types; the low types who were on the margin between buying the old good or none would not buy the new good. Under Hotelling, those consumers less disposed to the old good (located furthest from it) also are more attracted to the new good. This negative correlation of preferences permits a joint monopolist more latitude to raise the price of the old good and divert sales to the new good without causing some consumers to drop out of the market. To illustrate this, we contrast parts (a) and (b) of Greenstein and Ramey’s Lemma, which together yield their Proposition 2(a) that $G^m = G^c$, with the outcomes under Hotelling differentiation.\(^{16}\)

(a) Under vertical differentiation, if a monopolist over the old good adds the new good, total quantity and the price of the old good stay unchanged. In the Hotelling model, total quantity also is unchanged, but the price of the old good rises (recall Section 2). The types who got lower surplus from the old good (those located further) get higher surplus from the new good (are located closer to it), so adding the new good lets the monopolist raise the price of the old good and recapture all the diverted customers. Under vertical differentiation, raising the price of the old, low-quality good after the higher-quality good is added would shift some high types to the new good but would also cause some low types to drop out, which deters an increase in price.

(b) Under vertical differentiation, the quantity of the new good is the same whether the old good is supplied by the same monopolist or by competitive firms. In the Hotelling model, the quantity of the new good is higher under joint monopoly. The details are subtle (see Chen and Schwarz, 2010), but the basic idea can be seen by asking what happens if the monopolist over the new good acquires the previously competitive old good. Under Hotelling, the quantity of the new good will rise because the monopolist will profitably raise price more for the old good than for the new and maintain market coverage, whereas under vertical differentiation such unequal price increases would lower overall sales.

We have also analyzed the familiar model of a representative consumer with quadratic utility function over, and elastic demands for, two differentiated products (see Chen and Schwartz 2010, \(^{15}\)Under duopoly, they assume Cournot competition while we assume (differentiated) Bertrand. To isolate the role of the different product differentiation we compare monopoly to the competition regime (not duopoly) since the old good is then priced at marginal cost in both our setting and theirs.

\(^{16}\)Chen and Schwartz (2010) explain why GR’s Lemma implies that $G^m = G^c$, and provide further discussion of why the ensuing properties (a) and (b) hold under vertical differentiation but not in the Hotelling model.
Appendix B). Interestingly, the ranking of incentives to add the new product matches that in GR: $G^m = G^c < G^d$.\footnote{This differs from Spulber (2012, Proposition 4), because we assume $c_1 = c_2$, whereas Spulber’s result holds when $c_2$ is lower than $c_1$ and also $b$ is sufficiently low (high differentiation).} Despite the different preference structures and resulting demand systems, the representative consumer case shares two features with vertical — but not with Hotelling — differentiation. When a monopolist over the old good adds the new product, (a) the price of the old good is left unchanged; and (b) sales of the new good under joint monopoly are the same as in the alternative case where the old good is competitive.

This discussion suggests the following principle. When comparing the incentive of a monopolist to add a second product relative to the incentive of a more rivalrous firm, a key factor is the extent to which the monopolist can divert sales to the new product as opposed to leaking sales to outside goods if it raises the price of its old product.\footnote{This is related to the concept of upward pricing pressure from a horizontal merger between sellers of differentiated products (Farrell and Shapiro 2010). Starting at pre-merger equilibrium, if the seller of product $A$ merges with the seller of $B$, the merged firm’s incentive to raise the price of $A$ is proportional to the price-cost margin on $B$ and the diversion ratio—the fraction of $A$’s lost sales that will be recaptured by $B$. In our setting, a higher diversion ratio similarly increases the incentive of a monopolist over an initial good $A$ to raise $A$’s price after it adds product $B$. But our analysis focused on whether this pricing coordination ability can give an initial monopolist a stronger incentive to add product $B$ through innovation than for a firm that will compete against product $A$.}

5 Concluding Remarks

In contrast to Arrow’s famous result for process innovations, this paper showed that the incentive to invest in nondrastic product innovations could be higher for a secure monopolist (unconcerned with strategic preemption) than under alternative market structures where the innovating firm would face competition from the old good. While a monopolist’s incentive is diluted because a new substitute product would divert sales from its initial product, our analysis highlights an opposing effect: a monopolist can coordinate the pricing of the two products to increase profit from the new good.

A priori, either effect may dominate. An important factor is the extent to which an increase in the price of the old product will shift sales to the new product rather than drive consumers out of the market. This diversion ratio — and the ranking of product innovation incentives of a monopolist compared to a rivalrous firm — varies with the nature of product substitutability, as shown by the contrasting findings under our Hotelling differentiation versus Greenstein and Ramey’s (1998) for vertical differentiation.

Is there evidence that an exogeneous increase in market power can foster product innovation? Berry and Waldfogel’s (2001) analysis of radio mergers offers a potential test. The Telecommuni-
cations Act of 1996 allowed increased concentration of radio station ownership in local broadcast markets, prompting a major wave of radio mergers. If the higher concentration caused a significant increase in market power in enough local markets, the episode would offer a natural experiment for testing how market power affects product innovation incentives. Studying 243 local media markets from 1993 to 1997, Berry and Waldfogel (BW) found that increased concentration was associated with slower growth in the number of stations but faster growth in programming formats (e.g., the genre of music). One interpretation offered by BW is that a multi-station owner will space out its formats more than would separate owners to limit competition between its stations. However, this theory assumes that product locations do not affect stations’ advertising prices (BW fn.10); with endogenous prices, competing stations may differentiate their formats more so as to soften price competition (Tirole 1988, Anderson and Coate 2005). Our model provides an alternative interpretation of BW’s finding: reduced competition due to mergers enables higher advertising prices for the initial formats, making it profitable for a firm to add hitherto uneconomical formats that compete with the old formats. On the other hand, BW offer interesting evidence that the spacing decisions (hence number of formats) of joint owners were partly intended to preempt entry, as strategic consideration absent from our analysis which deliberately assumed a secure rather than entry-threatened monopolist.

Draganska et al. (2009) are closer to our framework since they analyze how a merger affects product choices abstracting from preemption motives. Studying the merger of two out of the top three manufacturers of “superpremium” ice cream, Dreyer’s and Nestlé, they analyze the number of vanilla varieties (products) offered in 64 U.S. regional markets. They simulate the profit-maximizing number of products pre merger and post merger. On average across markets, they predict the number of varieties will fall slightly. However, they note that the reverse would occur for sufficiently high fixed costs per flavor, because the reduction in price competition then would allow only a merged firm to cover the fixed cost of additional flavors. Since the authors

19 The caveat is needed because conceivably regulators may have calibrated the permissible mergers so as not to significantly weaken competition, for example, the permissible increase in concentration varied by local markets inversely to the initial concentration level.

20 Our model precludes an analysis of such location choices since the feasible product locations are assumed exogenous (at the ends of the Hotelling line). Gandhi et al. (2008) offer a theoretical analysis of post-merger product repositioning on the Hotelling line.

21 BW note that jointly owned stations in separate markets operate in similar formats more frequently than do randomly selected stations, a pattern consistent with economies of specialization, but the frequency of similar formats is even greater when the jointly owned stations are in the same local market, a pattern consistent with a strategic motive. Further, the format overlap between jointly owned local stations is greater in larger markets, where a smaller “hole” in products space would suffice to attract entry, than in smaller markets.

22 The U.S. Federal Trade Commission approved the merger in 2003 subject to divestitures. See <www.ftc.gov/opa/2003/06/nestle.shtm>
argue that fixed costs per flavor vary significantly across markets (e.g., advertising costs and slotting fees paid to supermarkets) and since the set of products (ice cream varieties) is chosen on a regional market basis, the merger conceivably would have increased the number of products in some markets absent the divestitures ordered by the FTC.\textsuperscript{23}

For future research, it would be interesting to identify natural experiments that control for other factors and exogenously generate different market structures, some involving a secure monopolist and others allowing product market rivalry, and compare the extent of product innovation. The natural experiments may involve an industry in a given country at different points in time (e.g., following a regulatory change) or across different countries. The regulatory differences that alter market structure could track some of the policy scenarios in the Introduction, for example, varying the breadth of patent protection or the stringency of merger policy.

\textsuperscript{23}In their survey of merger efficiencies claimed to the FTC Coate and Heimert (2009) include an expanded range of products. Typically, the merging parties will attribute this expansion to cost savings or other synergies and will not mention price increase on existing products as a contributing factor for obvious reasons. But an expanded set of products is also consistent with our price coordination effect undertaken by a merged firm.
References


