

Churn vs. Diversion: An Illustrative Model

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Abstract. An important question in merger analysis is how much of a firm’s lost output after a unilateral price increase will shift to the merger partner. To estimate this diversion ratio, antitrust agencies sometimes use data on consumer switching (“churn”), potentially caused by various reasons. This paper uses a tractable model of oligopoly competition to investigate the relation between churn and diversion, depending on what caused the churn. If the cause is an exogenous decrease in a firm’s product quality and all prices remain constant, or an increase in its marginal cost that induces a price increase only by that firm, then churn ratios will equal the corresponding diversion ratios; for the same quality or cost shocks, if churn is observed after all prices adjust to the new equilibrium, churn ratios will generally differ from diversion ratios, but nevertheless will still track the ranking of diversion ratios across the firm’s competitors. If the exogenous shock is an increase in a rival’s product quality, or a decrease in its cost that leads to a price decrease, the churn ratio to that rival will always overstate the diversion ratio. We also consider churn caused by shifts in consumer preferences, broadly interpreted to include changed circumstances or learning about product attributes. Plausibly, churn ratios can then suggest a wrong ranking of how intensely the firm competes with various rivals.

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1. INTRODUCTION

In evaluating the likely competitive effects of a merger between sellers of differentiated substitute products, a central question is the strength of competition between those products relative to alternatives. An increasingly accepted measure of the importance of, say, product 2 as a competitor to product 1 is the *diversion ratio* from 1 to 2: the fraction of unit sales lost by product 1 due to an increase in its price that would be diverted to product 2.¹ In a discrete choice context, if firm 1's price increase would cause it to lose 200 customers and firm 2 to gain 100 of them while firm 3 gains 50 and another 50 drop out, the diversion ratios to firms 2 and 3 are 50% and 25%, respectively, and it is natural to identify firm 2 as the closest competitor to firm 1.

When the competing products are in different physical units, diversion ratios will be less suited to assessing relative substitutability. However, as explained later, diversion ratios remain valuable for computing Upward Pricing Pressure (UPP) from a horizontal merger. In a merger of single-product firms 1 and 2, the UPP on product 1's price is the increased profit earned on product 2 per unit reduction in the sales of product 1. This constitutes an opportunity cost of selling product 2 that will be internalized by the merged firm but was ignored by firm 1 initially, thereby providing a post-merger incentive to raise the price of product 1 (and similarly for product 2). UPP has been forcefully advocated as a screen for anti-competitive mergers in differentiated product industries (Farrell and Shapiro, 2010; see also Moresi, 2010).

Diversion ratios, and the UPP concept that utilizes them, have been incorporated into the revised U.S. Horizontal Merger Guidelines (2010): "Diversion ratios between

¹As originally noted by Willig (1991), the diversion ratio from product j to product k depends on the cross-price elasticity of k with respect to j , product j 's own price elasticity, and the initial quantities. Shapiro (1996) introduced diversion ratios as a tool in horizontal merger analysis.

products sold by one merging firm and products sold by the other merging firm can be very informative for assessing unilateral price effects. ... The Agencies rely much more on the value of diverted sales than on the level of the HHI [Herfindahl-Hirschman Index] for diagnosing unilateral price effects in markets with differentiated products.” (*Id.*, section 6.1.) Diversion ratios and UPP have also been adopted in other jurisdictions, including the UK and European Union (Oldale and Padilla, 2013). Although diversion ratios and UPP are not intended as an alternative to full-blown merger simulation, they can be a useful initial screen in merger analysis. Moreover, diversion ratios have relevance beyond horizontal mergers, e.g. they can be used along with price-cost margins to analyze tax incidence under differentiated Bertrand competition (Weyl and Fabinger, 2013).

Diversion ratios for a single-product firm correspond to the consumer switching patterns following a unilateral price increase by that firm, while product characteristics, demand conditions, and rivals’ prices are held constant. This ideal experiment is rarely available, and in practice agencies sometimes use as an indicator the observed *churn ratios*: of the customers that left a particular firm for whatever reason, what fraction switched to each of its competitors.²

It is widely recognized, of course, that churn and diversion ratios can differ depending on the reasons for churn, as the Court in *H&R Block* eloquently noted.³ Nevertheless, and understandably, churn data is often used, based on a gut sense that

²The U.S. Department of Justice has relied on switching or churn data to be indicative of diversion when direct estimates of diversion are difficult to obtain. See, e.g. *U.S. v. H&R Block, Inc., et al.* (2011). The U.S. FCC staff reviewing the proposed merger of AT&T with T-Mobile estimated diversion ratios between wireless carriers based on the number of customers who switched, using data on the porting of telephone numbers (Kwerel, Lafontaine and Schwartz, 2012).

³“The IRS data, however, provides little direct insight about *why* any given taxpayer switched methods of preparation. The switch could have been for reasons of price, convenience, changes in the consumer’s personal situation, and increase or decrease in tax complexity, a loss of confidence in prior method of preparation, or any other reason. As opposed to switching, diversion refers to a consumer’s response to a measured increase in the price of a product.” (*U.S. vs. H&R Block et al.*, 2011, p. 35.)

it provides a (possibly crude) proxy for diversion. Ineed, the *Block* Court itself added: “The plaintiff’s expert argues, however, that the IRS switching data can provide at least some estimate of diversion. While this approach is not without its limitations, as discussed further below, the Court finds that the switching data is at least somewhat indicative of likely diversion ratios.” (*Id.*, p. 36.)

In this paper we go beyond the general observation that churn can differ from diversion, and address two questions. Depending on the reason for churn, what are the biases in estimating diversion ratios from churn data? And when do churn ratios, even if biased, rank a firm’s rivals correctly in order of competitive importance? We address these questions in a tractable oligopoly model, a variant on the spokes model of Chen and Riordan (2007). In practice, outside observers often have some knowledge of the exogenous shock(s) that directly or indirectly induced the churn, so our results may provide useful guidance in such cases.

Section 2 presents the model. Each firm competes with two rivals and each pair are connected by a Hotelling line to form a triangle. Consumers are located (only) on the three line segments, and a segment’s competitive importance depends on both the number of consumers located on it and the degree of product differentiation, the “transport cost” along that line. These parameters determine the diversion ratios.

In Section 3 we consider churn away from firm 1 (say) due to supply-side shocks. First, we analyze an exogenous decrease in its product quality. In our model this is formally equivalent to a price increase, so the churn ratios from firm 1 to its rivals will, by definition, equal the diversion ratios if rivals’ prices are held constant. However, we also compute churn ratios at the *new equilibrium prices*, characterize the quantitative biases in estimating diversion ratios, and show that churn ratios nevertheless will correctly identify the closer competitor (Proposition 1). We then consider churn away from firm 1 due to an increase in the product quality of a rival, say firm 2. If all prices are held constant, the churn from firm 1 to firm 2 obviously will overstate

the diversion ratio (because here only firm 2 becomes more attractive relative to firm 1, unlike for a unilateral price increase by firm 1); but we show that this bias persists even after prices adjust to the new equilibrium (Proposition 2). In addition, we show that Proposition 1 applies equally when the exogenous shock is an increase in firm 1's marginal cost instead of a decrease in quality, and likewise Proposition 2 applies to a decrease in firm 2's marginal cost instead of an increase in its quality.

In Section 4, we analyze churn due to changing preferences, broadly interpreted to include a change in a consumer's circumstances that alters the relative appeal of various products, or learning from experience that the chosen product failed to match expectations. We provide analytic conditions under which the churn ratio between a pair of firms overstates or understates the corresponding diversion ratio (Proposition 3). Relatedly, we show that the churn ratio from firm 1 to, say, firm 2 can be higher than to firm 3 even though firm 3 is the closer competitor to firm 1 (Proposition 4), and discuss scenarios where this wrong ranking could plausibly arise in practice.

Section 5 offers concluding remarks on the role of diversion ratios and UPP when competing products are in different units, and the relationship between diversion ratios and firms' market shares. The latter, instead of churn data, are sometimes used as a proxy for diversion ratios. In our model (at least in some tractable cases), the ranking of diversion ratios tracks market shares—the diversion ratio from firm 1 to firm 2 is higher than to firm 3 if and only if firm 2's market share is higher; but diversion to the larger rival will be disproportionately greater than its market share. We also briefly discuss limitations of our analysis and potential extensions.

2. THE MODEL

Consider a simple extension of the Hotelling model with three firms, 1, 2, and 3, that are pair-wise connected by three Hotelling lines of unit length forming a triangle, as shown in Figure 1. Consumers are uniformly distributed on the lines connecting

the three firms, l_{12} , l_{13} , and l_{23} , but the mass of consumers can differ across these segments. A consumer located on l_{12} at distance $x_{12} \in [0, 1]$ to firm 1 is denoted as consumer x_{12} , and similarly $x_{13} \in [0, 1]$ and $x_{23} \in [0, 1]$ denote consumers on l_{13} and l_{23} . To consume a product, each consumer must incur “transport costs” proportional to its distance from the relevant firm, where the transport cost parameter represents the degree of product differentiation between the two firms on a given segment. Each consumer desires at most one unit of the product, and values firm j ’s product at V_j minus transport costs, where j indexes each of the two firms on the relevant segment.

Insert Figure 1 here

Without loss of generality we analyze competition from the standpoint of firm 1, with its rivals firms 2 and 3. To simplify, we normalize to 1 the number of consumers and the unit transport cost on the segment l_{23} , but allow for asymmetries between l_{12} and l_{13} in both unit transport costs and the mass of consumers. Specifically, the unit transport cost is 1 on segment l_{12} and t on l_{13} , while the consumer populations are m and n respectively.

This simple model, adapted from Chen and Riordan’s (2007) spokes model,⁴ has several notable features. First, it describes a form of discrete choice demand where each consumer only has a first and a second preferred choice, and effectively chooses between these two alternatives in market equilibrium. The model is thus especially convenient to analyze, and can be easily extended to incorporate more firms.⁵ As suggested by Somaini and Einav (2013), the model and its extensions can also provide a useful framework for empirical analysis.

⁴Recent applications that use variants of the spokes model include, for example, Caminal (2010), Caminal and Granero (2012), Germanoa and Meier (2013), Rhodes (2011), and Reggiani (2014). Our formulation here is more closely related to the static version of the model developed by Somaini and Einav (2013) for studying dynamic competition.

⁵For instance, with four firms there would be a network of six Hotelling lines connecting the firms. With only three firms, our model is equivalent to the Hotelling circle model with equidistant firms, but the equivalence does not extend beyond three firms.

Second, it is a spatial model of product differentiation with non-localized competition, where each firm competes with every other firm in the market, but for different sets of consumers. The unit transport cost is a measure of product differentiation and, hence, $t < 1$ indicates less differentiation and more intense competition between firms 1 and 3 than between firms 2 and 1 or 2 and 3. To fix ideas, $t < 1$ is plausible if firms 1 and 3 both sell sports cars whereas firm 2 sells mini vans. Moreover, the number of consumers on a line can also be a measure of competition, so that $n > m$ indicates “more” direct competition between firms 1 and 3 than between 1 and 2.

Suppose for now that firms compete in the above setting for only one period, choosing prices p_1 , p_2 , and p_3 independently and simultaneously. We make the standard assumption that each firm sets a uniform price to all consumers. Thus, each firm’s equilibrium price will depend on competitive conditions in its two directly connected segments *and* in the third segment, because the latter affects the rivals’ equilibrium prices. The resulting expressions for equilibrium prices have closed form solutions but are complex. We defer them to the Appendix where we also discuss equilibrium market shares, and proceed here with the analysis of demand functions and diversion ratios, which do not require deriving equilibrium prices.

A consumer located between firms 1 and 3, say, obtains net surplus

$$U(x_{13}, p_1, p_3) = \begin{cases} (V_1 - p_1) - tx_{13} & \text{if buys product 1} \\ (V_3 - p_3) - t(1 - x_{13}) & \text{if buys product 3} \end{cases} \quad (1)$$

and similarly for consumers located on the other two segments. Note that all potential consumers of product j would obtain the same surplus $V_j - p_j$ before subtracting transport costs. We thus have the following Remark, that will prove helpful when analyzing the effects of changes in a firm’s quality:

Remark 1 *In our model, a decrease of size Δ in the valuation parameter V_j has the*

same effect on consumer demands as an increase of size Δ in firm j 's price.

Denote the marginal consumers on lines l_{12} , l_{13} , and l_{23} , by \hat{x}_{12} , \hat{x}_{13} , and \hat{x}_{23} respectively. Unless otherwise stated, we assume $V_j = V$ for $j = 1, 2, 3$, with V large enough that the market is fully covered in equilibrium. Then

$$\hat{x}_{12} = \frac{p_2 - p_1 + 1}{2}; \quad \hat{x}_{13} = \frac{p_3 - p_1 + t}{2t}; \quad \hat{x}_{23} = \frac{p_3 - p_2 + 1}{2}. \quad (2)$$

The demand functions for firms 1, 2, and 3 are respectively

$$Q_1 = m\hat{x}_{12} + n\hat{x}_{13}, \quad Q_2 = m(1 - \hat{x}_{12}) + \hat{x}_{23}; \quad Q_3 = n(1 - \hat{x}_{13}) + 1 - \hat{x}_{23}. \quad (3)$$

The diversion ratio from firm i to firm j is defined as $d_{ij} = -(\partial Q_j / \partial p_i) / (\partial Q_i / \partial p_i)$. Holding rivals' prices constant, if a price increase by firm i causes all its departing customers to switch to the two rivals rather than drop out of the market (as in our Hotelling setting with the market always covered), the diversion ratio is

$$d_{ij} = \frac{\partial Q_j / \partial p_i}{\partial Q_j / \partial p_i + \partial Q_k / \partial p_i}.$$

Focusing on a price change by firm 1,

$$\frac{\partial Q_2}{\partial p_1} = \frac{m}{2}; \quad \frac{\partial Q_3}{\partial p_1} = \frac{n}{2t},$$

so the diversion ratio from firm 1 to firm 2 is

$$d_{12} = \frac{\frac{m}{2}}{\frac{m}{2} + \frac{n}{2t}} = \frac{\frac{m}{n}}{\frac{m}{n} + \frac{1}{t}}. \quad (4)$$

Thus, the diversion ratio from 1 to 2 will decrease if the relative number of consumers choosing between firms 1 and 2 versus 1 and 3 ($\frac{m}{n}$) falls, or if product differentiation

between 1 and 2 relative to that between 1 and 3 ($\frac{1}{t}$) rises.⁶

Observe that we can write

$$d_{12} - d_{13} = \frac{mt - n}{mt + n}, \text{ hence} \quad (5)$$

$$\text{sign}(d_{12} - d_{13}) = \text{sign}\left(\frac{m}{1} - \frac{n}{t}\right) = \text{sign}\left(\frac{m}{n} - \frac{1}{t}\right), \quad (6)$$

where $m/1$ is the number of consumers adjusted by the transport cost (differentiation parameter) on the segment l_{12} between firms 1 and 2, while n/t is the analogue on the segment l_{13} between firms 1 and 3. From firm 1's standpoint, therefore, firm 2 is a more important competitor than firm 3 in the sense of a larger diversion ratio if (i) the number of consumers relative to the transport cost is larger on segment l_{12} than on l_{13} , or, equivalently, if (ii) the number of consumers on l_{12} relative to l_{13} exceeds the relative transport costs on these segments.

The parameters m , n , and t that determine diversion ratios are generally unobservable. However, one can often observe data on the switching of consumers between firms. Our interest is whether and how such churn data may inform us about the diversion ratio. Define the (outbound) churn ratio from firm i to firm j as

$$c_{ij} = \frac{\# \text{ of consumers switching from } i \text{ to } j}{\text{Total } \# \text{ of consumers switching away from } i}. \quad (7)$$

By definition, the diversion ratio equals the churn ratio when churn is caused by an increase in firm 1's price, holding constant other prices, all product attributes, and consumer preferences. However, the actual observed churn may be caused by various factors. In the next section we examine the relationship between diversion and churn when the latter is induced by supply-side shocks, allowing for adjustment to the new

⁶Notice that since $\partial Q_2/\partial p_1$ and $\partial Q_3/\partial p_1$ are both independent of prices, d_{12} measures the diversion ratio not just for a marginal increase in p_1 , but also for any increase in p_1 with which all three firms still produce positive outputs.

equilibrium prices. Then, in Section 4, we will consider churn due to changes in consumer preferences.

3. CHURN DUE TO SUPPLY-SIDE SHOCKS

Our formal analysis addresses churn caused by shocks to product attributes that alter the value for all consumers equally—“product quality,” represented by the parameter V_j . To simplify, we normalize marginal costs to zero, but will explain how the analysis extends straightforwardly for shocks to a firm’s (constant) marginal cost.

3.1 Decrease in Product 1’s Quality

Suppose firm 1’s product quality decreases by $\Delta > 0$, i.e. V_1 falls by Δ . From Remark 1, the effect on consumer demands is equivalent to an increase by Δ in firm 1’s price. At the initial prices p_1, p_2 , and p_3 , the marginal consumers on the three segments now become, using (2),

$$\hat{x}_{12} = \frac{-\Delta + p_2 - p_1 + 1}{2}; \quad \hat{x}_{13} = \frac{-\Delta + p_3 - p_1 + t}{2t}; \quad \hat{x}_{23} = \frac{p_3 - p_2 + 1}{2}, \quad (8)$$

with the demands Q_1, Q_2 , and Q_3 again given by (3).

Before any price change, since

$$\frac{\partial Q_2}{\partial \Delta} = \frac{m}{2} \text{ and } \frac{\partial Q_3}{\partial \Delta} = \frac{n}{2t},$$

the number of consumers switching from 1 to 2 and 3 induced by Δ will be $\frac{m}{2}\Delta$ and $\frac{n}{2t}\Delta$, respectively, and hence the churn ratio is:

$$c_{12} = \frac{\frac{m}{2}\Delta}{\frac{m}{2}\Delta + \frac{n}{2t}\Delta} = \frac{\frac{m}{n}}{\frac{m}{n} + \frac{1}{t}} = d_{12}.$$

Remark 2 *In our model, if churn is caused only by a decrease in firm 1’s quality and prices remain constant, the churn ratio c_{1j} will equal the diversion ratio d_{1j} .*

Thus, churn patterns caused by a quality decrease with all prices held constant will give exact measures of diversion ratios (as would an increase in that firm’s marginal cost that led to an increase in its price while holding all else constant).⁷ Though straightforward, this observation is useful because it helps identify additional natural experiments where churn may provide a good estimate of diversion. Empirically, a quality reduction could reflect shocks such as deterioration in firm 1’s customer service or, in the context of video distribution, a dispute between a video provider and a programmer that causes the provider to lose some programming (“blackout”). Such a quality reduction will provide a closer approximation to the effect of an exogenous (cost-induced) price increase by firm 1 the more uniform is its impact on all consumers, in particular, when it does not alter the relative appeal of products 2 and 3 as alternatives to product 1.

Suppose, however, that all prices adjust to their new equilibrium levels and the observed churn occurs at these new equilibrium prices. We address two questions: (i) When will the churn ratio c_{12} yields a biased measure of the diversion ratio d_{12} and what determines the direction of the bias? (ii) If there is a bias, do the churn ratios c_{12} and c_{13} nevertheless track the ranking of the diversion ratios d_{12} and d_{13} , and thereby still correctly identify which of firm 1’s rival is its “closer” competitor?

Proposition 1 *For churn that occurs after product 1’s quality decreases by Δ and at the new equilibrium prices:*

- (i) *(Bias)* $c_{12} = d_{12}$ if $d_{12} = d_{13}$, $c_{12} > d_{12}$ if $d_{12} < d_{13}$, and $c_{12} < d_{12}$ if $d_{12} > d_{13}$.
- (ii) *(Ranking)* $c_{12} = c_{13}$ if $d_{12} = d_{13}$, $c_{12} > c_{13}$ if $d_{12} > d_{13}$, and $c_{12} < c_{13}$ if $d_{12} < d_{13}$.

⁷If rivals’ prices remain constant, churn induced by a fall in firm 1’s quality V_1 will yield an exact measure of the diversion ratios even if firm 1’s price adjusts downwards, as long as $V_1 - p_1$ still falls.

All of our results are proved in the Appendix. Part (i) of Proposition 1 says that the observed churn ratio from firm 1 to firm 2 will equal the corresponding diversion ratio if firms 2 and 3 are equally important competitors to firm 1 ($d_{12} = d_{13}$), and will overstate or understate the diversion ratio if firm 2 is less or more important than firm 3. To understand the intuition for these biases, consider for instance the case $d_{12} < d_{13}$. The reduction in firm 1's quality will shift demand to firms 2 and 3 (even after firm 1's price reduction) and lead them to raise prices. If segment l_{12} is competitively less important than l_{23} , as required for $d_{12} < d_{13}$, then firm 2's price increase will be smaller than firm 3's, so firm 2 will attract a larger share of firm 1's departing customers than it would had rivals' prices stayed constant, as assumed in defining d_{12} . Thus, $c_{12} > d_{12}$ in this case.

Part (ii) states that, notwithstanding the possible biases, churn ratios will correctly identify the stronger competitor to firm 1. That is, although c_{12} may over- or underestimate d_{12} , the ranking of c_{12} and c_{13} will always mirror that of d_{12} and d_{13} . The churn ratio from firm 1 to firm 2 will be higher than to firm 3 if and only if firm 1 competes more "strongly" with firm 2 than with firm 3 in the sense of diversion ratios.

Proposition 1 will apply equally if the exogenous shock is an increase in firm 1's constant marginal cost instead of a decrease in its product quality, again measuring churn at the new equilibrium prices, due to the following property:

Remark 3 *In our model, an exogenous change of size Δ in firm j 's marginal cost c_j will have identical effects on equilibrium quantities demanded and, hence, on churn ratios, as a change $-\Delta$ in firm j 's quality V_j .*

To see the equivalence, consider without loss of generality firm 1, and write its demand and profit functions as

$$Q_1 = D_1(p_1 - V_1, p_2 - V_2, p_3 - V_3)$$

$$\Pi_1 = (p_1 - c_1)D_1(p_1 - V_1, p_2 - V_2, p_3 - V_3).$$

Define firm j 's "dollar margin," as $m_j \equiv p_j - c_j$, and firm j 's "quality-adjusted marginal cost" as $z_j \equiv c_j - V_j$. We can then rewrite firm 1's profit function as

$$\Pi_1 = m_1 D_1(m_1 + z_1, m_2 + z_2, m_3 + z_3).$$

Thus, our model where firms are described by two parameters (c_j and V_j) and choose prices (p_j) is identical to a model where firms are described by a single parameter ($z_j \equiv c_j - V_j$) and choose margins ($m_j \equiv p_j - c_j$). Therefore, an increase in firm 1's marginal cost from c_1 to $c_1 + \Delta$ is equivalent to a decrease in firm 1's quality from V_1 to $V_1 - \Delta$. Either shock has the same effect on z_1 and, hence, on equilibrium margins and quality-adjusted prices $p_j^* - V_j (= m_j^* + z_j)$, which determine consumer demands.

3.2 Increase in a Rival's Product Quality

We now consider the switching patterns from firm 1 in response to an exogenous improvement in the offering of one of its rivals, say firm 2, represented by an increase of $\Delta > 0$ in firm 2's "product quality." After firm 2's quality increase, and holding prices constant at their initial levels, p_1, p_2 , and p_3 , the marginal consumers on the three segments are given in (2) become

$$\hat{x}_{12} = \frac{-\Delta + p_2 - p_1 + 1}{2}; \quad \hat{x}_{13} = \frac{p_3 - p_1 + t}{2t}; \quad \hat{x}_{23} = \frac{\Delta + p_3 - p_2 + 1}{2}, \quad (9)$$

with the demands Q_1, Q_2 , and Q_3 again given by (3). At the original prices, there will be consumers switching from 1 to 2, but not from 1 to 3, implying $c_{12} = 1 > d_{12}$. Thus, at constant prices following the quality change, the churn ratio from firm 1 to firm 2 will overstate the diversion ratio. This is obvious, since a quality increase for firm 2 at constant prices will reduce the appeal of firm 1 only relative to firm 2,

whereas a unilateral price increase by firm 1 and holding everything else constant—the experiment underlying d_{12} —will reduce firm 1’s appeal also relative to firm 3.

What can be said about churn patterns, however, if churn occurs after all prices adjust to the new equilibrium following firm 2’s quality increase? The possible complication arises if the price differential $p_3 - p_1$ declines, as this would cause firm 1 to lose some consumers also to firm 3, thereby reducing the churn ratio c_{12} below 1. When would this occur, and could c_{12} then potentially exceed the diversion ratio d_{12} ?

Proposition 2 *When churn from firm 1 is caused by an increase in firm 2’s product quality and churn occurs after all prices adjust to the new equilibrium, then*

$$c_{12} = \begin{cases} 1 & \text{if } m \geq 1 \\ \frac{(n+t)(3n+2mt)}{-mn^2+2mt^2+3nt+4n^2+2mnt} < 1 & \text{if } m < 1 \end{cases}.$$

Furthermore, it is always true that $c_{12} > d_{12}$.

By Remark 3, Proposition 21 would apply equally if the exogenous shock were a decrease in firm 2’s marginal cost rather than an increase in its quality.

Turning to the increase in firm 2’s quality, the role of $m \geq 1$ versus $m < 1$ is understood as follows. Since transport costs were assumed equal (to 1) on the segments between firms 2 and 1 and between firms 2 and 3 (l_{12} and l_{23}), segment l_{12} is competitively more important than l_{23} if and only if its mass of consumers is larger, $m \geq 1$ (recall Figure 1). In that case, the increase in firm 2’s quality will, in the new equilibrium, induce a (weakly) larger price reduction by firm 1 than by firm 3, due to the greater importance of l_{12} to firm 1 than of l_{23} to firm 3. The price differential $p_3 - p_1$ then expands, hence no customers switch from firm 1 to firm 3, leaving the churn ratio c_{12} equal to 1. If $m < 1$, firm 1’s equilibrium price reduction is larger than firm 3’s, so $p_3 - p_1$ declines and firm 1 loses some customers also to firm 3, causing

$c_{12} < 1$.⁸

Nevertheless, when the exogenous shock is an improvement in firm 2's quality, in this model we always obtain $c_{12} > d_{12}$: the churn ratio will overstate the diversion ratio even after the equilibrium price adjustments. One might have conjectured that if segment l_{12} were sufficiently unimportant relative to l_{23} , then the price responses to firm 2's quality increase could reduce the differential $p_3 - p_1$ sufficiently to drive c_{12} below d_{12} . However, the factor that makes l_{12} relatively less important—a lower mass of consumers m —will also reduce the diversion ratio d_{12} , ensuring $c_{12} > d_{12}$.

4. CHURN DUE TO CHANGING PREFERENCES

Consider customer switching driven not by a change in prices or product attributes but by an exogenous “change in preferences.” This can reflect a change in circumstances that alters the relative appeal of various products (e.g., a sports car becomes a poorer fit than a mini van to a new parent), or learning from experience that the chosen product performed worse than expected (e.g., driving a low-riding sports car next to trucks proved terrifying).

We represent such churn using a two-period extension of the static model, where in each period firms play the same static pricing game, but for some consumers their first and second preferred choices may change between periods. A simple way of modeling this is to assume that in the second period, a portion α_{12} of consumers on l_{12} switch locations between $x_{12} < 1/2$ and $x_{12} > 1/2$, while a portion α_{13} of consumers on l_{13} switch locations between $x_{13} < 1/2$ and $x_{13} > 1/2$. (For our purposes, we do not need to specify the switching pattern on l_{23} .) The consumer populations remain uniformly distributed on the lines in the second period. Therefore, equilibrium prices and market shares in both periods are the same as in the static model.⁹

⁸The role of all three segments in determining the equilibrium price changes following an exogenous shock was illustrated also in the discussion of Proposition 1

⁹Our reduced-form modeling of “changes in preferences” (here, locations) is obviously coarse and

The portion of consumers that will actually switch between firms is determined as follows. Let \hat{x}_{1j}^* , given explicitly in (14) and (15) in the Appendix, denote firm 1's equilibrium share of the market segment l_{1j} between firm 1 and firm j , $j = 2, 3$. Recall that \hat{x}_{1j}^* is also the location of the consumer who is indifferent between firms 1 and j . To switch from firm 1 to firm j , a consumer's preferences (i.e., location) must satisfy two conditions: (i) in the first period, $x_{1j} \in [0, \hat{x}_{1j}^*)$, so this consumer bought from firm 1; and (ii) in the second period, $x_{1j} \in (\hat{x}_{1j}^*, 1]$, so this consumer will buy from firm j .¹⁰ The portion of all consumers whose preferences change from $x_{1j} < 1/2$ to $x_{1j} > 1/2$ and, hence, who *potentially* will switch, is $\frac{1}{2}\alpha_{1j}$. The portion that will actually switch from firm 1 to firm j is therefore given by¹¹

$$\lambda_{1j} = \alpha_{1j} \min \{ \hat{x}_{1j}^*, 1 - \hat{x}_{1j}^* \}, \text{ for } j = 2, 3. \quad (10)$$

One can view λ_{1j} as the portion of consumers whose preferences change from 1 to j , adjusted by firm 1's share of consumers on the market segment l_{1j} (hereafter, "segment-share"). With equal shares, $\hat{x}_{1j}^* = \frac{1}{2}$, implying $\lambda_{1j} = \alpha_{1j}/2$.¹² With unequal shares, $\lambda_{1j} < \alpha_{1j}/2$. The *mass* of consumers who will switch from firm 1 to firm 2 and firm 3 is $m\lambda_{12}$ and $n\lambda_{13}$, respectively.

The churn ratio from firm 1 to 2 is therefore

$$c_{12} = \frac{m\lambda_{12}}{m\lambda_{12} + n\lambda_{13}} = \frac{1}{1 + \frac{n}{m} \frac{\lambda_{13}}{\lambda_{12}}}. \quad (11)$$

restrictive. The model can be extended to asymmetric changes in preferences between two firms, but retaining symmetry will greatly simplify the analysis, enabling us to focus on the relevant issues.

¹⁰The marginal consumer \hat{x}_{1j}^* can buy from either 1 or j . Since there is a continuum of consumers on l_{1j} , our analysis is not affected by from whom \hat{x}_{1j}^* makes the purchase.

¹¹If $\hat{x}_{1j}^* \leq \frac{1}{2}$, the portion that switch is $\frac{1}{2}\alpha_{1j}(\frac{\hat{x}_{1j}^*}{1/2}) = \alpha_{1j}\hat{x}_{1j}^*$, where $\frac{\hat{x}_{1j}^*}{1/2}$ is the fraction of potential switchers that initially bought from firm 1, since consumers are uniformly distributed on $[0, 1]$. If $\hat{x}_{1j}^* > \frac{1}{2}$, the portion that switch is $\frac{1}{2}\alpha_{1j}(\frac{1-\hat{x}_{1j}^*}{1/2}) = \alpha_{1j}(1 - \hat{x}_{1j}^*)$, where $\frac{1-\hat{x}_{1j}^*}{1/2}$ is the fraction of potential switchers that will buy from firm j .

¹²It is easily verified that segment shares will be equal, i.e. $\hat{x}_{12}^* = \hat{x}_{13}^* = \frac{1}{2}$, if $t = 1$ or if $m = n = 1$.

From (4), the diversion ratio from firm 1 to firm 2 is

$$d_{12} = \frac{\frac{m}{2}}{\frac{m}{2} + \frac{n}{2t}} = \frac{1}{1 + \frac{n}{m} \frac{1}{t}}. \quad (12)$$

The churn and diversion ratios are positively correlated through the relative number of consumers, n/m , on segments l_{13} and l_{12} . The ranking of c_{12} versus d_{12} , however, does not directly depend on n/m , since an increase in this term will decrease both c_{12} and d_{12} .¹³ The ranking is determined by $\lambda_{13}/\lambda_{12}$ and $1/t$: the ratio of the (segment-share adjusted) portion of consumers that change preferences between 1 and 3 relative to that between 1 and 2, and the relative product differentiation on the two segments (inversely related to transport costs). Comparing (11) and (12), $c_{12} = d_{12}$ when $\lambda_{13}/\lambda_{12} = 1/t$, and when these ratios are unequal, we have:

Proposition 3 *Suppose churn is caused only by changes in preferences. Then $c_{12} > d_{12}$ if $\lambda_{12}/\lambda_{13} > t/1$, and $c_{12} < d_{12}$ if $\lambda_{12}/\lambda_{13} < t/1$. That is, the churn ratio will over-estimate the diversion ratio if the (segment-share adjusted) portion of consumers changing their preferred brand between 1 and 2 relative to the portion changing between 1 and 3 is higher than the product differentiation between 1 and 3 relative to the differentiation between 1 and 2. If the inequality is reversed, the churn ratio will under-estimate the diversion ratio.*

The condition for churn to over-state diversion, $\lambda_{12}/\lambda_{13} > t/1$, is more likely to hold when $\lambda_{12}/\lambda_{13}$ is higher, which raises the churn ratio c_{12} without affecting the diversion ratio d_{12} ; or when $t/1$ is lower, which lowers d_{12} without affecting c_{12} . Three special cases are:

(i) Suppose $t = 1$: equal product differentiation between 1 and 3 as between 1 and 2. Then firm 1 in equilibrium obtains half the customers on the line segment

¹³However, when $t \neq 1$, m and n might affect c_{12} indirectly, by inducing unequal equilibrium prices and, hence, a segment share $\hat{x}_{12}^* \neq 1/2$ and $\lambda_{1j} \neq \alpha_{1j}/2$.

with either rival, $\hat{x}_{1j}^* = \frac{1}{2}$, hence the proportion who switch from firm 1 to rival j is $\lambda_{1j} = \frac{\alpha_{1j}}{2}$, from (10). Therefore, $c_{12} > d_{12}$ if $\lambda_{12}/\lambda_{13} = \alpha_{12}/\alpha_{13} > 1$, so c_{12} will over-estimate d_{12} if a higher proportion of consumers change their preferred product between 1 and 2 than between 1 and 3 ($\alpha_{12} > \alpha_{13}$).

(ii) Suppose $m = n = 1$: equal numbers of consumers. Then, $\hat{x}_{12}^* = \frac{1}{2} \frac{2t+3}{3t+2}$ from (14) in the Appendix, $\lambda_{12} = \frac{1}{2} \frac{4t+1}{3t+2} \alpha_{12}$, $\lambda_{13} = \alpha_{13}/2$, and c_{12} over-estimates d_{12} if

$$\frac{\alpha_{12}}{\alpha_{13}} > \frac{3t+2}{4t+1}t,$$

which, for $t \leq 1$, holds if $\frac{\alpha_{12}}{\alpha_{13}} > 1$, or $\alpha_{12} > \alpha_{13}$.

(iii) Suppose $\lambda_{12} \geq \lambda_{13}$: the (segment-share adjusted) portion of consumers changing their preferred product between 1 and 2 is at least as high as that between 1 and 3. Then c_{12} over-estimates d_{12} if product differentiation is weaker between 1 and 3 than between 1 and 2 ($t < 1$).

In a subset of the cases where the churn ratio from firm 1 to firm 2 overstates the corresponding diversion ratio ($c_{12} > d_{12}$), comparing churn ratios may wrongly suggest that firm 2 is firm 1's closer competitor. Specifically:

Proposition 4 (*wrong ranking*) *If $\frac{t}{1} < \frac{n}{m} < \frac{\lambda_{12}}{\lambda_{13}}$, then: (i) competition is weaker between products 1 and 2 than between 1 and 3 in the sense that the diversion ratio is lower ($d_{12} < d_{13}$), and yet (ii) the churn ratio is higher between 1 and 2 than between 1 and 3 ($c_{12} > c_{13}$).*

The logic is as follows. From Proposition 3, $c_{12} > d_{12}$ if $t/1 < \lambda_{12}/\lambda_{13}$, which is independent of the relative number of consumers n/m on the segments l_{12} and l_{13} . Diversion ratios, on the other hand, are independent of $\lambda_{12}/\lambda_{13}$, and $d_{12} < d_{13}$ will hold if $t/1 < n/m$ (from (6)). Churn ratios are independent of the relative differentiation $t/1$, and $c_{12} > c_{13}$ will hold if $n/m < \lambda_{12}/\lambda_{13}$. Thus, for $\frac{n}{m} \in (\frac{t}{1}, \frac{\lambda_{12}}{\lambda_{13}})$

we will have both $d_{12} < d_{13}$ and $c_{12} > c_{13}$.¹⁴

Discussion

We now discuss informally how the counter-intuitive possibility shown in Proposition 4, that churn between products 1 and 2 can exceed that between 1 and 3 even though the latter are stronger substitutes (less differentiated), may arise in practice.

Consider an industry, such as automobiles, whose products are classified into distinct *segments*, such as sports cars and mini vans. Products within a segment are differentiated along various attributes, such as customer service.¹⁵ Products in different segments exhibit additional sources of differentiation, that we label *performance dimensions*; for example, sports cars provide thrills but lack the cargo room or high ride of mini vans. If the performance dimensions are sufficiently important to consumers, there will be greater differentiation, hence weaker substitution, between products in different segments than within a segment.¹⁶

Now consider “changes in preferences.” One such possibility is a change in consumer circumstances. Then it is quite plausible that churn will be greater between sellers of more dissimilar products. A sports car driver expecting children may well switch to a mini van rather than another sports car; and when the children are grown, the mini van owner may revert to a sports car. As another example of changed circumstances, consumers whose income rises over the life cycle often migrate from lower-quality

¹⁴Given $t/1 < \lambda_{12}/\lambda_{13}$, as needed for $c_{12} > d_{12}$, if n/m lies outside the interval $(t/1, \lambda_{12}/\lambda_{13})$, the ranking of churn ratios and diversion ratios will coincide. If $n/m < t/1$, then $d_{12} > d_{13}$ and $c_{12} > c_{13}$; if $n/m > \lambda_{12}/\lambda_{13}$, then $d_{12} > d_{13}$ and $c_{12} > c_{13}$.

¹⁵In automobiles, much of the customer service is performed by independent dealers. A customer nonetheless may hold the firm partly responsible for any mishaps because it approved the dealers. Also, a customer may blame the firm for disputes over ambiguities regarding, for example, the breadth of coverage of a warranty.

¹⁶In our Hotelling structure, suppose products 1 and 3 are in the same segment (sports cars, or DBS video) while 2 is in a different segment (mini vans, or terrestrial video). The “transport cost” between 1 and 3 will reflect the importance consumers attach to intra-segment attributes. The transport cost between 1 and 2 will, additionally, reflect inter-segment attributes. If the latter are sufficiently important, the transport cost is likely to be higher between 1 and 2.

versions of a product to higher-quality versions instead of switching among lower-quality versions.

A more complex scenario involves uncertainty over product attributes, leading to ex post mistakes—a mismatch between a customer’s preferences and the chosen product. If uncertainty about inter-segment attributes is minor, then one would expect greater churn between products in the same segment than across segments. A sports car driver who is disappointed with the customer service from its vendor is more likely to switch to another sports car than to a mini van. On the other hand, if uncertainty over inter-segment attributes is large, one could see greater churn across product segments than within. A sports car driver who learns that the low ride on highways is more taxing than expected or that the cargo room is simply too small is more likely to switch to a mini van than to another sports car.

In short, the theoretical possibility shown in Proposition 4, that churn data may wrongly identify a firm’s closer competitor, can plausibly arise if changes in consumer circumstances are prevalent or if across-segment uncertainty is large compared to within-segment. We discuss two potential examples beyond automobiles.

First, consider churn between the two satellite radio providers, XM and Sirius, prior to their merger. Salop et al. (2007) cite data (redacted from the public version of their submission to the FCC) showing that a minority of subscribers who disconnect from one satellite radio service churn to the other, with the majority switching to other audio entertainment products such as free terrestrial radio. Even if XM and Sirius were each other’s closest competitors, this pattern is consistent with our changing preferences scenario, *if* much of the switching was prompted by a change in the person’s circumstances, e.g. a sharp reduction in income that induced a switch to free terrestrial radio, or a realization that the added features of satellite radio were not worth the subscription price.¹⁷

¹⁷However, Salop et al. provide other evidence besides switching patterns for why substitutability

Next, consider competition between video distributors in a given locality in the US: two firms that use satellite transmission (DBS) nationally, and a rival that uses terrestrial transmission (the local cable company, and in some areas also the local phone company). One expects the DBS firms to compete more closely with each other than with a terrestrial provider, because the alternative technologies can have different performance characteristics and because the DBS firms predominantly sell video on a stand-alone basis whereas terrestrial providers predominantly sell video bundled with broadband Internet access. However, if customer switching is observed over a period where product attributes and prices are stable, it would not be surprising to see more switching from a DBS firm to a terrestrial rival. The switching may reflect “changing circumstances,” such as increased need for broadband, which makes the terrestrial provider’s bundled offering more attractive. The switching may also reflect disappointment with the DBS provider. If the culprit was customer service, switching is more likely to the DBS rival. But if the culprit was technology—e.g. satellite reception in bad weather was worse than expected—then switching is more likely to a terrestrial provider: “you didn’t like satellite, so try something different.” But, importantly, the pool of customers who switch in these scenarios is not representative of the switching patterns that would occur in response to a unilateral price increase.¹⁸

between XM and Sirius may be weak, including significant content differentiation and the need to invest in a new radio.

¹⁸A similar criticism, the “Silent Majority Fallacy,” has been made of the Elzinga-Hogarty test for geographic market definition in antitrust. The test finds a given region overly narrow to qualify as an antitrust market—i.e. a hypothetical monopolist comprised of all the sellers of the relevant product in that region would not profitably raise price by a significant amount—if a sufficient fraction of all consumers residing in that region travel outside it to purchase the product (e.g. patients travelling to out-of-region hospitals). However, this argument presumes “that the non-traveling ‘silent majority’ is similar to the traveling (pre-merger) minority and is protected against a post-merger price increase by those patents poised to join those already willing to migrate.” (Elzinga and Swisher, 2011.)

5. CONCLUDING REMARKS

In our model, all competitors' outputs are measured in common units, the number of customers. Comparing output-based diversion ratios becomes less meaningful when outputs are in different units. (E.g. suppose the firms offer substitute forms of entertainment: firm 1 is a theatre, firm 2 is a sports stadium, and firm 3 sells DVDs.) This problem of units cannot always be ignored, since diversion ratios are geared to assessing competition among sellers of *differentiated* products, which quite naturally may involve different output units.¹⁹ A potential alternative is to use diversion ratios based on dollar sales rather than physical units, but this too has its problems.²⁰

Alternative measures of substitution, however, arguably have even greater flaws (Werden, 1998). For example, cross-elasticities are in percentages, and therefore do not indicate the absolute increase in sales captured by various competitors in response to a price increase by the initial firm (e.g., a higher cross-elasticity of demand for good 2 than for good 3 with respect to the price of good 1 could merely reflect a lower initial sales volume for good 2).²¹ Moreover, as noted in the Introduction, diversion ratios are not only used to gauge substitutability but also serve as an input for computing the Upward Pricing Pressure (UPP) from a merger. The UPP on the product of firm 1, say, due to a merger with the supplier of product 2 is given by $UPP_1 = d_{12}(p_2 - c_2)$, where d_{12} is the diversion ratio from 1 to 2, and $p_2 - c_2$ is the dollar margin between 2's price and marginal cost. Because d_{12} expresses additional output sold of product

¹⁹This units problem also implies that the diversion ratio from firm 1 to firm 2 can exceed 100%, although this point is sometimes overlooked when describing the diversion ratio as the fraction of firm 1's lost sales that was diverted to firm 2.

²⁰For instance, suppose the dollar diversion ratios are $d_{12} = 80\%$ and $d_{13} = 40\%$ (where the sum can exceed 100% due to the different units and different prices), but $p_2 = 10p_3$. Then of the customers who left firm 1, five times as many switched to firm 3 than to firm 2, yet the dollar diversion ratio is twice as large to firm 2 due entirely to its higher price.

²¹The diversion ratio to product 2 from product 1 equals $(\eta_{21}Q_2)/(\eta_{11}Q_1)$, where η_{jk} is the price-elasticity of demand for product j with respect to the price of k , and Q_i is the sales quantity of product i (Willig, 1991).

2 per unit lost by 1, and $p_2 - c_2$ is in dollars per unit of product 2, UPP_1 measures dollars gained on product 2 per unit lost of product 1. Diversion ratios, therefore, can be used for computing UPP regardless of whether products 1 and 2 are measured in the same physical units. As a practical matter, diversion ratios and UPP are likely to see continued use. In a simple model, we identified the relationship between diversion ratios and observed churn data depending on the specific reasons for churn.

Besides churn, competition agencies often can observe market shares (perhaps imperfectly), but lack the information needed to estimate the demand parameters that determine diversion ratios. Willig (1991) showed, *inter alia*, that with logit demand, diversion ratios are proportional to rivals' market shares, e.g. if firm 2's market share is twice as large as firm 3's, then a unilateral price increase by firm 1 will cause twice as much diversion to firm 2 as to firm 3. He goes on to caution that this proportionality relation need not hold with other demand systems. In that vein, we now discuss briefly the relation between market shares and diversion ratios in our model.

For the general asymmetric case, the expressions for equilibrium prices, outputs and market shares are complex functions of the demand parameters m, n , and t . However, the expressions are simple in two cases: when all segments exhibit equal product differentiation ($t = 1$) or equal numbers of consumers ($m = n = 1$). For both cases, diversion ratios and (aggregate) market shares are related as follows. Among firm 1's rivals: (i) their market shares (s_2 and s_3) and diversion ratios from firm 1 have the same ranking; and (ii) diversion to the larger rival will be disproportionately greater than its market share. For example, if $s_2 > s_3$ then $d_{12} > d_{13}$, with $d_{12}/d_{13} > s_2/s_3$ (unlike the equality for logit demand). Thus, the larger rival's market share understates its relative importance as a competitor to firm 1.

The proof of this result and further discussion can be found in the Appendix (Proposition 5), but the basic intuition runs as follows. The diversion ratio from firm 1 to firm 2 reflects the importance of competition between those firms relative to compe-

tition between 1 and 3, i.e. the relative competitive importance of market segments l_{12} and l_{13} . In our model, at least in the cases above, this also determines the ranking of aggregate market shares for firms 2 and 3. Those aggregate market shares, however, reflect also conditions on the segment l_{23} , on which firms 2 and 3 are otherwise symmetric. Thus, the difference in the market shares of firm 1’s competitors will be diluted relative to the difference in their diversion ratios.

An important message from our analysis is the need to recognize the reason for customer switching. We identified the quantitative biases in using switching data to estimate diversion ratios when switching from a firm is caused by supply-side shocks—a decrease in its quality or increase in its marginal cost, or the opposite shocks to a rival—after incorporating the adjustment of equilibrium prices. When, instead, switching is due to “changing preferences,” we stressed that switching patterns might quite plausibly yield even the wrong qualitative ranking of the closer competitors. Correspondingly, an agency seeking to estimate diversion ratios from switching data should seek natural experiments where a firm’s quality fell or its cost rose (ideally without changes in rivals’ prices); and it should be very cautious about making inferences from switching data observed over a period with no known supply shocks, as such switching might well reflect changes in preferences.

Turning to limitations of our analysis and potential extensions, our modeling of changing preferences was highly stylized. For example, the fraction of consumers who change their preferences (locations on a Hotelling line) was assumed symmetric and independent of product differentiation.²² More refined modeling could disaggregate “changing preferences” into changed circumstances or learning about product quality. Another extension would be to allow more than three firms, as in the spokes

²²As a counter-example, suppose firms 1 and 2 distribute video programming via satellite while firm 3 uses cable. Consider customer switching caused by learning that satellite reception during rain is worse than expected. This could be represented as an equal reduction in the qualities of goods 1 and 2 for the relevant consumers. The extent of switching to firm 3 (only) would depend also on the “transport costs” on the segment between each of those firms and firm 3.

model of Chen and Riordan (2007). The relationships between diversion ratios and churn ratios following supply-side shocks to a firm and the ensuing price adjustments (Propositions 1 and 2) would then depend on the properties of the additional market segments, because equilibrium prices depend on *all* segments. Finally, in our model each consumer only chooses between two products. While this assumption is obviously unrealistic, it may not be overly problematic for purposes of estimating diversion ratios, because the pattern of switching in response to a firm's unilateral price increase will depend only on the second preference of each of its customers. That said, our model is only intended to be illustrative and we do not wish to overstate its direct empirical applicability.

APPENDIX

A1. Equilibrium Prices in the Model of Section 2

We normalize the production cost of all firms to zero. Thus, prices should be interpreted as markups. For any firm i , $i = 1, 2, 3$, its profit function is $\pi_i = p_i Q_i$, where the demand functions Q_i were given in (3). Equilibrium prices are determined by the solutions to the three first-order conditions $\partial\pi_i/\partial Q_i = Q_i + p_i\partial Q_i/\partial p_i = 0$ or

$$\begin{aligned} m\frac{p_2-p_1+1}{2} + n\frac{p_3-p_1+t}{2t} - p_1\left(\frac{m}{2} + \frac{n}{2t}\right) &= 0, \\ m\left(1 - \frac{p_2-p_1+1}{2}\right) + \frac{p_3-p_2+1}{2} - p_2\frac{m+1}{2} &= 0, \\ n\left(1 - \frac{p_3-p_1+t}{2t}\right) + \left(1 - \frac{p_3-p_2+1}{2}\right) - p_3\left(\frac{n}{2t} + \frac{1}{2}\right) &= 0, \end{aligned} \quad (13)$$

from which we obtain the following equilibrium prices:

$$\begin{aligned} p_1^* &= \frac{t}{2} \frac{3n + 6mn^2 + 6m^2n + 9mn + 3nt + 6n^2 + 5mnt + 6mt + 6m^2t}{3mn^2 + 3mt^2 + 3m^2t^2 + 3nt + 3n^2 + 3m^2nt + 7mnt}, \\ p_2^* &= \frac{1}{2} \frac{3mn^2 + 6mt^2 + 3n^2t + 6m^2t^2 + 6nt + 3n^2 + 4mnt^2 + 3mn^2t + 6m^2nt + 10mnt}{3mn^2 + 3mt^2 + 3m^2t^2 + 3nt + 3n^2 + 3m^2nt + 7mnt}, \\ p_3^* &= \frac{t}{2} \frac{6n + 6mn^2 + 3m^2n + 6m^2t + 9mn + 6mt + 6n^2 + 3m^2nt + 5mnt}{3mn^2 + 3mt^2 + 3m^2t^2 + 3nt + 3n^2 + 3m^2nt + 7mnt}. \end{aligned}$$

Furthermore, firm 1's equilibrium share on the market segments l_{12} and l_{13} are respectively

$$\hat{x}_{12}^* = \frac{\left(9mn^2 + 6mt^2 - 3nt^2 - 3n^2t + 6m^2t^2 + 9nt\right) + 9n^2 - mnt^2 - 3mn^2t + 6m^2nt + 15mnt}{4(3mn^2 + 3mt^2 + 3m^2t^2 + 3nt + 3n^2 + 3m^2nt + 7mnt)}, \quad (14)$$

$$\hat{x}_{13}^* = \frac{3n + 6mn^2 - 3m^2n + 6mt^2 + 6m^2t^2 + 3nt + 6n^2 + 9m^2nt + 14mnt}{4(3mn^2 + 3mt^2 + 3m^2t^2 + 3nt + 3n^2 + 3m^2nt + 7mnt)}. \quad (15)$$

It is then straightforward to check that $\hat{x}_{13}^* = \frac{1}{2}$ if $t = 1$ or if $m = n = 1$. On the other hand, $\hat{x}_{12}^* = \frac{1}{2}$ if $t = 1$ but $\hat{x}_{12}^* = \frac{1}{2} \frac{2t+3}{3t+2}$ if $m = n = 1$.

The following comparative statics can be easily verified:

(i) A marginal increase in t raises p_i^* for each firm $i = 1, 2, 3$. (ii) A marginal increase in m raises all p_i^* if $t < 1$, lowers all p_i^* if $t > 1$, and does not affect any equilibrium price if $t = 1$. (iii) A marginal decrease in n raises all p_i^* if $t < 1$, lowers all p_i^* if $t > 1$, and does not affect any equilibrium price if $t = 1$.

The intuition behind these comparative statics is straightforward. First, although a higher t softens price competition directly only between firms 1 and 3, uniform pricing causes these firms' prices to rise also when competing against firm 2, whose price also will rise since prices here are strategic complements. Second, if $t < 1$, the market segment l_{13} is more competitive than the other two segments and, hence, a relatively smaller number of consumers on this segment (a lower n or a higher m) will soften overall competition, leading to higher prices for all firms (whereas a higher n or a lower m will reduce all prices if $t > 1$).

A2. New Equilibrium After Decrease in Firm 1's Product Quality

After Firm 1's quality decreases by Δ , the new equilibrium prices, which satisfy the first-order conditions

$$\begin{aligned} m \frac{-\Delta + p_2 - p_1 + 1}{2} + n \frac{-\Delta + p_3 - p_1 + t}{2t} - p_1 \left(\frac{m}{2} + \frac{n}{2t} \right) &= 0 \\ m \left(1 - \frac{-\Delta + p_2 - p_1 + 1}{2} \right) + \frac{p_3 - p_2 + 1}{2} - p_2 \left(\frac{m}{2} + \frac{1}{2} \right) &= 0 \\ n \left(1 - \frac{-\Delta + p_3 - p_1 + t}{2t} \right) + \left(1 - \frac{p_3 - p_2 + 1}{2} \right) - p_3 \left(\frac{n}{2t} + \frac{1}{2} \right) &= 0 \end{aligned}$$

are given by

$$\begin{aligned}
p_1^* &= \frac{6mt^2+3nt^2+6n^2t+6m^2t^2+3nt+5mnt^2+6mn^2t+6m^2nt+9mnt-\Delta(2mn^2+3mt^2+2m^2t^2+3nt+2n^2+2m^2nt+6mnt)}{6mn^2+6mt^2+6m^2t^2+6nt+6n^2+6m^2nt+14mnt}, \\
p_2^* &= \frac{3mn^2+6mt^2+3n^2t+6m^2t^2+6nt+3n^2+4mnt^2+3mn^2t+6m^2nt+10mnt+\Delta(n+mt)(n+2mn+2mt)}{6mn^2+6mt^2+6m^2t^2+6nt+6n^2+6m^2nt+14mnt}, \\
p_3^* &= \frac{6mt^2+6n^2t+6m^2t^2+6nt+5mnt^2+6mn^2t+3m^2nt+3m^2nt^2+9mnt+\Delta(2n+2mn+mt)(n+mt)}{6mn^2+6mt^2+6m^2t^2+6nt+6n^2+6m^2nt+14mnt}.
\end{aligned} \tag{16}$$

Substituting these prices into $m(1 - \hat{x}_{12})$ and $n(1 - \hat{x}_{13})$, we obtain

$$\begin{aligned}
\frac{\partial[m(1-\hat{x}_{12})]}{\partial\Delta} &= \frac{1}{4} \frac{(n+t)m(2m+3)(n+mt)}{3mn^2+3mt^2+3m^2t^2+3nt+3n^2+3m^2nt+7mnt} \\
\frac{\partial[n(1-\hat{x}_{13})]}{\partial\Delta} &= \frac{1}{4} \frac{(m+1)n(n+mt)(2n+3t)}{t(3mn^2+3mt^2+3m^2t^2+3nt+3n^2+3m^2nt+7mnt)}.
\end{aligned} \tag{17}$$

The numbers of consumers switching from firm 1 to firms 2 and 3 are, respectively, $\frac{\partial[m(1-\hat{x}_{12})]}{\partial\Delta} \Delta$ and $\frac{\partial[n(1-\hat{x}_{13})]}{\partial\Delta} \Delta$. The churn ratio from firm 1 to 2 is therefore

$$c_{12} = \frac{(n+t)m(2m+3)t}{2mn^2+3mt^2+2m^2t^2+3nt+2n^2+2m^2nt+6mnt}. \tag{18}$$

It follows that

$$c_{12} - d_{12} = \frac{(n-mt)mnt}{(n+mt)(2mn^2+3mt^2+2m^2t^2+3nt+2n^2+2m^2nt+6mnt)},$$

which takes the sign of $(n - mt)$, and hence, from (5), the sign of $-(d_{12} - d_{13})$. This proves part (i) of Proposition 1.

Turning to part (ii), from (17) we have,

$$c_{13} = \frac{(m+1)n(2n+3t)}{2mn^2+3mt^2+2m^2t^2+3nt+2n^2+2m^2nt+6mnt}. \tag{19}$$

Thus, from (18) and (19):

$$c_{12} - c_{13} = \frac{(-n+mt)(2n+3t+2mn+2mt)}{2mn^2+3mt^2+2m^2t^2+3nt+2n^2+2m^2nt+6mnt},$$

which takes the sign of $-n + mt$. Since $d_{12} - d_{13} = \frac{-n+mt}{n+mt}$, it takes the opposite sign of $c_{12} - c_{13}$. This proves part (ii) of Proposition 1.

Finally, to show that Proposition 1 applies equally to an increase in firm 1's marginal cost as to a decrease in its quality, rewrite the marginal costs of firm j as c_j , with initially all $c_j = 0$. Now suppose c_1 increases by Δ . Then, with all $V_j = V$, the first-order conditions for the new equilibrium prices become

$$\begin{aligned} m \frac{p_2 - p_1 + 1}{2} + n \frac{p_3 - p_1 + t}{2t} - (p_1 - \Delta) \left(\frac{m}{2} + \frac{n}{2t} \right) &= 0, \\ m \left(1 - \frac{p_2 - p_1 + 1}{2} \right) m \left(1 - \frac{p_2 - p_1 + 1}{2} \right) + \frac{p_3 - p_2 + 1}{2} - p_2 \left(\frac{m}{2} + \frac{1}{2} \right) &= 0, \\ n \left(1 - \frac{p_3 - p_1 + t}{2t} \right) + \left(1 - \frac{p_3 - p_2 + 1}{2} \right) - p_3 \left(\frac{n}{2t} + \frac{1}{2} \right) &= 0. \end{aligned}$$

Solving the equilibrium prices and substituting them into \hat{x}_{12} and \hat{x}_{13} , we have

$$\frac{\partial (m(1 - \hat{x}_{12}))}{\partial \Delta} = \frac{1}{4} \frac{(n+t)m(2m+3)(n+mt)}{3mn^2 + 3mt^2 + 3m^2t^2 + 3nt + 3n^2 + 3m^2nt + 7mnt},$$

$$\frac{\partial (n(1 - \hat{x}_{13}))}{\partial \Delta} = \frac{1}{4} \frac{(m+1)n(n+mt)(2n+3t)}{t(3mn^2 + 3mt^2 + 3m^2t^2 + 3nt + 3n^2 + 3m^2nt + 7mnt)}.$$

Hence

$$\begin{aligned} c_{12} &= \frac{\frac{\partial [m(1-\hat{x}_{12})]}{\partial \Delta} \Delta}{\frac{\partial [m(1-\hat{x}_{12})]}{\partial \Delta} \Delta + \frac{\partial [n(1-\hat{x}_{13})]}{\partial \Delta} \Delta} = \frac{(n+t)m(2m+3)t}{2mn^2 + 3mt^2 + 2m^2t^2 + 3nt + 2n^2 + 2m^2nt + 6mnt}, \\ c_{13} &= \frac{\frac{\partial [n(1-\hat{x}_{13})]}{\partial \Delta} \Delta}{\frac{\partial [m(1-\hat{x}_{12})]}{\partial \Delta} \Delta + \frac{\partial [n(1-\hat{x}_{13})]}{\partial \Delta} \Delta} = \frac{(m+1)n(2n+3t)}{2mn^2 + 3mt^2 + 2m^2t^2 + 3nt + 2n^2 + 2m^2nt + 6mnt}, \end{aligned}$$

which are identical to those from (18) and (19). Therefore, the signs of $c_{12} - d_{12}$ and $c_{12} - c_{13}$ are the same as when the churn was due to a reduction of V_1 by Δ .

A3. New Equilibrium after Increase in Firm 2's Product Quality

After Firm 2's quality increases by Δ , the new equilibrium prices, which satisfy the first-order conditions

$$\begin{aligned} m \frac{-\Delta + p_2 - p_1 + 1}{2} + n \frac{p_3 - p_1 + t}{2t} - p_1 \left(\frac{m}{2} + \frac{n}{2t} \right) &= 0 \\ m \left(1 - \frac{-\Delta + p_2 - p_1 + 1}{2} \right) + \frac{\Delta + p_3 - p_2 + 1}{2} - p_2 \left(\frac{m}{2} + \frac{1}{2} \right) &= 0 \\ n \left(1 - \frac{p_3 - p_1 + t}{2t} \right) + \left(1 - \frac{\Delta + p_3 - p_2 + 1}{2} \right) - p_3 \left(\frac{n}{2t} + \frac{1}{2} \right) &= 0 \end{aligned}$$

are given by

$$\begin{aligned} p_1^* &= \frac{6mt^2 + 3nt^2 + 6n^2t + 6m^2t^2 + 3nt + 5mnt^2 + 6mn^2t + 6m^2nt + 9mnt - t\Delta(m+1)(n+2mn+2mt)}{6mn^2 + 6mt^2 + 6m^2t^2 + 6nt + 6n^2 + 6m^2nt + 14mnt} \\ p_2^* &= \frac{3mn^2 + 6mt^2 + 3n^2t + 6m^2t^2 + 6nt + 3n^2 + 4mnt^2 + 3mn^2t + 6m^2nt + 10mnt + \Delta(3mn^2 + 2mt^2 + 2m^2t^2 + 2nt + 3n^2 + 2m^2nt + 6mnt)}{6mn^2 + 6mt^2 + 6m^2t^2 + 6nt + 6n^2 + 6m^2nt + 14mnt} \\ p_3^* &= \frac{6mt^2 + 6n^2t + 6m^2t^2 + 6nt + 5mnt^2 + 6mn^2t + 3m^2nt + 3m^2nt^2 + 9mnt - t\Delta(m+1)(2n+mn+2mt)}{6mn^2 + 6mt^2 + 6m^2t^2 + 6nt + 6n^2 + 6m^2nt + 14mnt} \end{aligned} \quad (20)$$

Substituting these prices into the \hat{x}_{12} and \hat{x}_{13} , we obtain, at the new equilibrium,

$$\begin{aligned} \frac{\partial(m(1-\hat{x}_{12}))}{\partial\Delta} &= \frac{1}{4} \frac{(n+t)(3n+2mt)(m+1)}{3mn^2 + 3mt^2 + 3m^2t^2 + 3nt + 3n^2 + 3m^2nt + 7mnt} \\ \frac{\partial(n(1-\hat{x}_{13}))}{\partial\Delta} &= \frac{1}{4} \frac{(m+1)n^2(1-m)\Delta}{3mn^2 + 3mt^2 + 3m^2t^2 + 3nt + 3n^2 + 3m^2nt + 7mnt} \end{aligned} \quad (21)$$

The number of consumers switching from 1 to 2 is thus

$$m(1-\hat{x}_{12})|_{\Delta} - m(1-\hat{x}_{12})|_{\Delta=0} = \int_0^{\Delta} \frac{\partial(m(1-\hat{x}_{12}))}{\partial\Delta} d\Delta = \frac{\partial(m(1-\hat{x}_{12}))}{\partial\Delta} \Delta$$

The number of consumers switching from 1 to 3 is 0 if $m \geq 1$ and

$$n(1-\hat{x}_{13})|_{\Delta} - n(1-\hat{x}_{13})|_{\Delta=0} = \int_0^{\Delta} \frac{\partial(n(1-\hat{x}_{13}))}{\partial\Delta} d\Delta = \frac{\partial(n(1-\hat{x}_{13}))}{\partial\Delta} \Delta \text{ if } m < 1.$$

If $m < 1$, then there are consumers switching from 1 to both 2 and 3, with

$$c_{12} = \frac{\frac{\partial[n(1-\hat{x}_{13})]}{\partial\Delta}}{\frac{\partial[m(1-\hat{x}_{12})]}{\partial\Delta} + \frac{\partial[n(1-\hat{x}_{13})]}{\partial\Delta}} = \frac{(n+t)(3n+2mt)}{-mn^2 + 2mt^2 + 3nt + 4n^2 + 2mnt}.$$

Therefore, since $d_{12} = \frac{\frac{m}{n+1}}{\frac{m}{n} + \frac{1}{t}}$, if $m \geq 1$, $c_{12} - d_{12} > 0$, and if $m < 1$,

$$c_{12} - d_{12} = n \frac{2mt^2 + 3nt + 3n^2 + m^2nt + mnt}{(n+mt)(-mn^2 + 2mt^2 + 3nt + 4n^2 + 2mnt)} > 0.$$

A4. Changes in Preferences

Proof of Proposition 4:

$$d_{12} < d_{13} \iff \frac{\frac{m}{2}}{\frac{m}{2} + \frac{n}{2t}} < \frac{\frac{n}{2t}}{\frac{m}{2} + \frac{n}{2t}} \iff \frac{1}{1 + \frac{n}{m} \frac{1}{t}} < \frac{1}{1 + \frac{m}{n} \frac{t}{1}} \iff \frac{t}{1} < \frac{n}{m}, \text{ and}$$

$$c_{12} > c_{13} \iff \frac{m\lambda_{12}}{m\lambda_{12} + n\lambda_{13}} > \frac{n\lambda_{13}}{m\lambda_{12} + n\lambda_{13}} \iff \frac{1}{1 + \frac{n}{m} \frac{\lambda_{13}}{\lambda_{12}}} > \frac{1}{1 + \frac{m}{n} \frac{\lambda_{12}}{\lambda_{13}}} \iff \frac{n}{m} < \frac{\lambda_{12}}{\lambda_{13}}.$$

Thus, for $\frac{n}{m} \in \left(\frac{t}{1}, \frac{\lambda_{12}}{\lambda_{13}}\right)$ we will have both $d_{12} < d_{13}$ and $c_{12} > c_{13}$.

A5. Diversion Ratios and Market Shares

Proposition 5 *Assume that all market segments exhibit either: 1) equal product differentiation ($t = 1$, but m and n may differ); or 2) equal numbers of consumers ($m = n = 1$, but t may differ from 1). Then, among firm 1's competitors:*

(i) their market shares track the ranking of diversion ratios from firm 1

$$s_2 = s_3 \iff d_{12} = d_{13}; \quad s_2 < s_3 \iff d_{12} < d_{13}; \quad \text{and } s_2 > s_3 \iff d_{12} > d_{13} \quad (22)$$

(ii) The relative market shares and relative diversion ratios are related as follows:

$$\frac{s_2}{s_3} = \frac{d_{12}}{d_{13}} \text{ if } s_2 = s_3; \quad \frac{s_2}{s_3} < \frac{d_{12}}{d_{13}} \text{ if } s_2 > s_3; \quad \text{and } \frac{s_2}{s_3} > \frac{d_{12}}{d_{13}} \text{ if } s_2 < s_3 \quad (23)$$

Part (ii) of Proposition 5 says that if firm 2's aggregate market share is larger than firm 3's, then 2's market share relative to 3's will understate 2's diversion ratio from firm 1 *relative* to 3's (whereas under logit demand, $s_2/s_3 = d_{12}/d_{13}$).

Consider first the intuition in Case 1: $t = 1$. All firms will then charge the same equilibrium prices, $p_1^* = p_2^* = p_3^*$: on any given segment, the two firms' *preferred* price for that segment depends only on the differentiation parameter, and with $t = 1$ the preferred prices are equal for all three segments. Given equal prices, firms have equal shares (1/2) of each segment, so the aggregate market shares of firm 1's rivals, s_2 and s_3 , will depend only on the number of consumers m and n on the segments l_{12} and l_{13} , and the same holds for diversion ratios (given $t = 1$). The ranking of diversion ratios therefore tracks aggregate market shares — part (i) of Proposition 5. However, the disparity in diversion ratios d_{12} and d_{13} depends only on the difference in the number of consumers on segments l_{12} versus l_{13} (m vs. n); whereas the aggregate market shares of firms 2 and 3 (s_2 and s_3) depend also on segment l_{23} , where customers are split equally, which dilutes the discrepancy in aggregate shares compared to diversion ratios, thereby explaining part (ii) of Proposition 5.

Now turn to Case 2: $m = n = 1$. This time, any asymmetry between firms 2 and 3 is determined by the product differentiation on the segments l_{12} and l_{13} (rather than the number of consumers on those segments). If differentiation between firms

1 and 3 is lower than between 1 and 2 ($t < 1$), then: (i) The diversion ratio from firm 1 to 2 will be smaller than from 1 to 3 ($d_{12} < d_{13}$), and firm 2's market share will be smaller than firm 3's — because 2's equilibrium price will be higher (due to softer competition between 2 and 1 on l_{12} than between 3 and 1 on l_{13}), so firm 2 will capture less than half the segment l_{23} between it and firm 3. But (ii) the ratio of the diversion ratios will exceed that of market shares because the latter is determined by the price difference $|p_2^* - p_3^*|$, which is attenuated since these prices depend also on the segment l_{23} where demand conditions are symmetric.

We now provide the proof of Proposition 5 for the two cases.

Case 1. $t = 1$: *equal product differentiation between all three pairs of firms.*

Substituting the prices $p_1^* = p_2^* = p_3^* = 1$ in the demand expressions yields the equilibrium outputs for the three firms:

$$Q_1 = \frac{m+n}{2}; \quad Q_2 = \frac{m+1}{2}; \quad Q_3 = \frac{n+1}{2}.$$

The market shares of the three firms are respectively

$$\begin{aligned} s_1 &= \frac{\frac{m+n}{2}}{\frac{m+n}{2} + \frac{m+1}{2} + \frac{n+1}{2}} = \frac{1}{2} \frac{m+n}{m+n+1}, \\ s_2 &= \frac{\frac{m+1}{2}}{\frac{m+n}{2} + \frac{m+1}{2} + \frac{n+1}{2}} = \frac{1}{2} \frac{m+1}{m+n+1}, \\ s_3 &= \frac{\frac{n+1}{2}}{\frac{m+n}{2} + \frac{m+1}{2} + \frac{n+1}{2}} = \frac{1}{2} \frac{n+1}{m+n+1}. \end{aligned}$$

Thus, firm 2's market share relative to firm 3's share is given by

$$\frac{s_2}{s_3} = \frac{m+1}{n+1}. \quad (24)$$

The diversion ratios from firm 1 are related as follows

$$\frac{d_{12}}{d_{13}} = \frac{\frac{m}{2}}{\frac{n}{2t}} = \frac{m}{n}, \text{ given } t = 1. \quad (25)$$

Both parts of Proposition 5 follow immediately from comparing the above expressions.

Case 2. $m = n = 1$: *equal numbers of consumers on all market segments.* Then,

$$p_1^*|_{m=n=1} = \frac{5t}{3t+2}; \quad p_2^*|_{m=n=1} = \frac{4t+1}{3t+2}; \quad p_3^*|_{m=n=1} = \frac{5t}{3t+2}.$$

The equilibrium outputs for the three firms are

$$\begin{aligned} Q_1 &= \frac{\frac{4t+1}{3t+2} - \frac{5t}{3t+2} + 1}{2} + \frac{\frac{5t}{3t+2} - \frac{5t}{3t+2} + t}{2t} = \frac{5t+1}{2(3t+2)}, \\ Q_2 &= \left(1 - \frac{\frac{4t+1}{3t+2} - \frac{5t}{3t+2} + 1}{2}\right) + \frac{\frac{5t}{3t+2} - \frac{4t+1}{3t+2} + 1}{2} = \frac{4t+1}{3t+2}, \\ Q_3 &= \left(1 - \frac{\frac{5t}{3t+2} - \frac{5t}{3t+2} + t}{2t}\right) + \left(1 - \frac{\frac{5t}{3t+2} - \frac{4t+1}{3t+2} + 1}{2}\right) = \frac{5t+1}{2(3t+2)}. \end{aligned}$$

The market shares of the three firms are respectively

$$\begin{aligned} s_1 &= \frac{\frac{5t+1}{2(3t+2)}}{\frac{5t+1}{2(3t+2)} + \frac{4t+1}{3t+2} + \frac{5t+1}{2(3t+2)}} = \frac{5t+1}{6(3t+2)}, \\ s_2 &= \frac{\frac{4t+1}{3t+2}}{\frac{5t+1}{2(3t+2)} + \frac{4t+1}{3t+2} + \frac{5t+1}{2(3t+2)}} = \frac{1(4t+1)}{3(3t+2)}, \\ s_3 &= \frac{\frac{5t+1}{2(3t+2)}}{\frac{5t+1}{2(3t+2)} + \frac{4t+1}{3t+2} + \frac{5t+1}{2(3t+2)}} = \frac{5t+1}{6(3t+2)}. \end{aligned}$$

Thus, firm 2's market share relative to firm 3's share is now given by

$$\frac{s_2}{s_3} = \frac{2(4t+1)}{5(t+1)}. \quad (26)$$

The relative diversion ratios from firm 1 are now simply

$$\frac{d_{12}}{d_{13}} = \frac{\frac{m}{2}}{\frac{n}{2t}} = t, \text{ given } m = n. \quad (27)$$

Part (i) of Proposition 5 follows since $\frac{s_2}{s_3} = 1$ for $t = 1$ and $\frac{s_2}{s_3}$ increases in t . Part (ii) follows since

$$\frac{d_{12}}{d_{13}} - \frac{s_2}{s_3} = \frac{t5(t+1) - 2(4t+1)}{5(t+1)} = \frac{t[2 + 5(t-1)] - 2}{5(t+1)} > (=) < 0 \text{ as } t > (=) < 1.$$

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Figure 1

