

## COMPATIBILITY INCENTIVES OF A LARGE NETWORK FACING MULTIPLE RIVALS\*

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Under network effects, we analyze when a firm with the largest market share of installed-base customers prefers incompatibility with smaller rivals that are themselves compatible. With incompatibility, consumers realize that intra-network competition makes the rivals' network more aggressive than a single-firm network in adding customers. Consequently, under incompatibility the unique equilibrium can entail tipping *away* from the largest firm whatever its market share. The largest firm is more likely to prefer incompatibility as its share rises (above fifty per cent is necessary) or the potential to add consumers falls; the number of rivals and strength of network effects have ambiguous implications.

### I. INTRODUCTION

INDUSTRIES DISPLAYING POSITIVE NETWORK EFFECTS—the value to a user of the good rises with the number of consumers who use compatible versions of that good—are ubiquitous.<sup>1</sup> Network effects can be direct, as in communications services where the very purpose is to contact other users. They also can be indirect, as when a larger user base elicits lower prices or greater variety of complements, typically due to scale economies in their supply. Examples of indirect network effects include; the hardware-software paradigm, where a base good is consumed with variable amounts of complementary products, such as a computer's hardware or operating

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<sup>1</sup>The economics literature on markets with network effects is by now voluminous. For recent surveys, see Farrell and Klemperer [2004], Koski and Kretschmer [2004], and Gandal [2002]. Some key earlier contributions are Rohlfs [1974], David [1985], Katz and Shapiro [1985], Besen and Johnson [1986], Farrell and Saloner [1986], Arthur [1989], Church and Gandal [1992], Economides [1996a], and Matutes and Régibeau [1996].

system and its application programs; and 'two-sided markets' (Armstrong [2004], Rochet and Tirole [2004]), where greater participation by each end-user group encourages greater participation by the other, as in payment cards where more cardholders foster more merchant acceptance and vice versa.

In order to tap more fully the benefits from larger networks, some degree of cooperation among competitors is needed—whether active (e.g., establishing interconnection facilities) or passive (e.g., refraining from blocking rivals' access to software interfaces). A long-standing regulatory and competition concern in network markets is that a firm with a large enough share of the industry's customer base may, even at a cost to itself, impede competitors' sharing in those network effects so as to strengthen its *relative* 'quality' position. Impediments can entail imposing above-cost variable charges for network access (e.g., inflated call termination fees) or contrived technical and other non-price impediments to compatibility. Such concerns prompted mandated interconnection in the early days of the U.S. telephone industry under threat of antitrust action (the 1913 Kingsbury Commitment; see Brock [1994]), mandates reaffirmed in the U.S. Telecommunications Act of 1996 and similar laws elsewhere.

The same policy concern is manifest in major 'new economy' network industries—the Internet and computer software. Two prominent cases involved attempted mergers of Internet backbone providers, firms that supply high-capacity transmission links to smaller Internet service providers and large businesses. The largest backbone, WorldCom, sought to merge with its largest competitor, MCI and later Sprint. The core concern of European and U.S. competition authorities—hotly contested by the merging parties—was that the merged entity would command such a large share of the Internet customer base that it might gain by degrading, or refraining from upgrading, its interconnection with smaller rivals, or might use this threat to impose asymmetric interconnection charges. Unlike traditional circuit-switched telephony, interconnection of internet backbone was unregulated and traditionally provided at reciprocal zero charges. Attempting to maintain good interconnection by introducing regulation could be problematic, because the cost of access might be raised or its lowered in various non-price ways ('sabotage' as termed by Beard, Kaserman, and Mayo [2001]). Instead, U.S. and European competition authorities elected to forestall the risk of worsened interconnection incentives by blocking the largest backbone from expanding its customer base through merger.<sup>2</sup>

<sup>2</sup>The MCI/WorldCom merger was concluded in 1998 subject to divestiture of MCI's Internet operations, while MCI WorldCom/Sprint was abandoned in 2000 under pressure from the European Commission and U.S. Department of Justice. See European Commission [1998, 2000], U.S. Department of Justice [2000] and WorldCom and Sprint [2000]. Valued at

Concerns with sharing of direct network effects also surfaced in the acquisition by American Online (AOL) of Time Warner (Faulhaber [2002]). AOL had about 40% of U.S. Internet subscribers and refused to interoperate its leading text-based Instant Messenger (IM) system with competitors, citing concerns with security and privacy. While the merger would not alter AOL's subscriber share, control of Time Warner's cable broadband assets might enhance AOL's *ability* to deny interoperability with future data-intensive IM services that require access to cable broadband.<sup>3</sup> A key question for the FCC was whether AOL's subscriber share was sufficiently large to create an incentive to restrict compatibility for exclusionary motives.

Finally, the landmark antitrust case, *U.S. v. Microsoft*, centered on *indirect* network effects. There is remarkable consensus among commentators inclined to support the case (Dunham [2004], Gilbert and Katz [2001]) and those disposed against it (Klein [2001]) about the core of the Justice Department's case: Microsoft's attacks on Netscape's Internet browser, Navigator, were in large part aimed at protecting the Windows operating system (OS) against potential rivals by restricting their access to application written for windows—preserving the 'applications barriers to entry.' (The divergence among commentators, and between Microsoft and the government, is largely over whether Microsoft's conduct crossed the permissible line of competition on the merits.) Windows' large user base spawned a rich supply of complementary application programs. Sun's Java programming language, in conjunction with a browser, threatened to become a 'middleware' layer of software between applications and the OS—applications would interface with Java—and thus function like an 'adapter' that allows OS competitors to share Windows applications. Microsoft feared that Navigator would become a strong distribution platform for Java and thereby help erode the applications barrier to entry.

The risk in the above and similar interventions, of course, is that the same mergers or practices can in principle arise for a host of legitimate business reasons, which policy makers should be reluctant to second guess absent a credible risk of significant exclusion. It is therefore important to gain a deeper understanding of when impeding compatibility for exclusionary reasons is less or more plausible, depending on industry characteristics that are observable or potentially can be estimated: the number of rivals, the strength of network effects, the scope for market expansion relative to the

over \$125 billion, the WorldCom/Sprint transaction would have been larger than any prior merger. For useful background on the Internet, see Cave and Mason [2001].

<sup>3</sup>To address this fear, the U.S. Federal Communications Commission (FCC), as a condition for the merger, barred AOL from offering broadband-based IM services until it resolved certain concerns with interoperability for such services. The FCC refrained from ordering AOL to offer interoperability with its *existing* text-based IM, judging that for these services the merger did not alter AOL's incentive and ability to restrict compatibility.

installed base, and the leading firm's share of the installed base. Such an analysis can potentially help in evaluating ambiguous compatibility-related conduct by the largest network or mergers that would create or expand the largest network.

We consider the following economic environment. *First*, there are no inter-firm payments for compatibility. Such payments sometimes are precluded by regulation (e.g., telephone interconnection charges are set near marginal cost) for fear they might facilitate exclusion or collusion. Paying for compatibility can also be impractical for contractual reasons, such as assigning fault if compatibility is inadequate (e.g., attempting to ensure that third-party applications written for the dominant platform are compatible with a rival platform). *Second*, compatibility requires the consent of both sides, it cannot be achieved unilaterally through 'adapters' or 'converters.' *Third*, compatibility is a zero-one choice, and entails no difference in cost or performance (beyond yielding broader network access). In practice, compatibility can add delay, raise cost, or harm performance, due to the need to coordinate across firms and to standardize product designs on a lower common denominator.<sup>4</sup> To simplify the analysis, we treat compatibility as neutral for cost and performance.<sup>5</sup> But we caution that by leaving out inherent technological constraints or other potential losses from compatibility, our model is not designed to address welfare questions immediately; it takes an intermediate step by investigating when incompatibility by the largest network can be profitable solely for exclusionary reasons.

For the most part, we also assume (A1) that a firm's compatibility choices cannot be targeted across rivals but must be uniform (as may occur when compatibility entails choosing an open standard), and (A2) that the smaller rivals are themselves compatible. Thus, most of our analysis will compare two potential regimes: *full compatibility* among all firms versus *autarky by the largest firm*, firm 1, which chooses to be incompatible with smaller compatible rivals. This is a useful starting point for addressing antitrust concerns with dominant-firm conduct in network industries, and some of the insights are relevant also for analyzing other compatibility regimes.

Our basic model, presented in Section II, is an extension of Crémer, Rey and Tirole [2000; henceforth, 'CRT'], who adapted Katz and Shapiro [1985] to incorporate installed-base customers. We extend CRT in two directions. First, firm 1, may face any number of smaller rivals. Second, we allow

<sup>4</sup>The reverse can also arise, for example, where compatibility would be achieved by adopting a common, off-the-shelf standard, while incompatibility would require devoting extra effort to developing a proprietary standard.

<sup>5</sup>Katz and Shapiro [1985] incorporate fixed costs of achieving compatibility. They show, for reasons that are by now familiar, that compatibility then can be socially excessive (fixed costs are spent partly to divert profit from rivals) or insufficient (consumers' gain from compatibility is not fully internalized).

network effects to be strong enough that incompatibility could produce tipping—all new customers eventually gravitate to one network—a central policy concern in network industries. Section III identifies the possible output-market equilibria conditional on firm 1's choosing autarky and demonstrates that our two extensions are complementary: autarky by firm 1 can yield a unique equilibrium with tipping *away* from firm 1, no matter how large its share of the industry customer base, if and *only if* firm 1 faces at least two rivals. The driving force is *intra-network competition*: consumers will expect a network of competing firms to act more aggressively in adding new customers, thereby expanding network quality, than would a single-firm network that is unable to commit not to exercise market power in the future.

Section IV analyzes the profitability to firm 1 of choosing autarky. An installed-base share of at least 50% is necessary but not sufficient to make autarky unambiguously profitable. An increase in this share encourages autarky by firm 1 (i.e., makes it profitable for a larger set of other parameter values), as does a higher common marginal cost or a larger industry installed base (both effects reduce the scope for market expansion *relative* to the installed base). However, the effect of increasing the number of rivals is ambiguous outside of tipping regions, and the role of stronger network effects is ambiguous more generally.

Section V uses the analysis to illustrate how to construct market-share safe harbors for the largest firm below which autarky is not clearly profitable, knowing only the number of rivals and the estimated industry growth rate under full compatibility. Section VI presents an environment in which uniform compatibility policies (assumption (A1) above) arise endogenously and, under a mild belief restriction, the rivals indeed prefer compatibility among themselves (validating assumption (A2)) if their installed bases are fairly symmetric. It also briefly reports on possibilities that arise under alternative compatibility regimes, when rivals have different installed bases or when *targeted* compatibility is possible. Section VII concludes.

## II. THE MODEL AND ALTERNATIVE COMPATIBILITY REGIMES

### II(i). *Installed Bases and Competition for New Customers*

Firms differ only in their locked-in, installed bases of customers. These customers will not switch to other firms, and pricing to them is set by prior contracts.<sup>6</sup> Competition is for new consumers. A new consumer whose type

<sup>6</sup> Forøs, Kind and Sand [2003] extend CRT's model by allowing the price paid by installed-base customers to increase in the size of that firm's network. We discuss this issue in Section VII. Ennis [2002] assumes that all customers are locked in, and that a large network bargains with smaller ones over payment for compatibility. He finds that payment will flow from the smaller to the larger network if and only if consumers' value for network effects is concave in the number of users. See also Besen, Milgrom, Mitchell and Srinagesh [2001].

is  $\tau$  and who buys firm  $i$ 's product at price  $p_i$  obtains net benefit  $\tau + vL_i - p_i$ , where  $\tau$  is that consumer's stand-alone valuation for the product (equal across firms),  $v > 0$  is a common parameter reflecting how intensely new consumers value network effects, and  $L_i$  (for 'links') is firm  $i$ 's effective network size—the number of new plus installed-base customers in the market that use products compatible with that of firm  $i$ .<sup>7</sup> We will refer to  $L_i$  as the 'quality' of firm  $i$ 's network. For simplicity, we treat compatibility between any two firms, denoted  $\theta$ , as either 0 or 1, where 1 means that each firm's customers enjoy equally good access to the total network consisting of both firms' customers, and 0 means that each firm's customers are inaccessible to customers of the other firm.<sup>8</sup>

Since consumers have no inherent preference for one firm or another ( $\tau$  is independent of which firm is chosen), any two firms  $i, j$ , that attract new customers in a consistent-expectations equilibrium must offer equal quality-adjusted prices:

$$(1) \quad p_i - vL_i = p_j - vL_j \equiv \bar{\tau}.$$

Price differences therefore are proportional to differences in network size:  $p_i - p_j = v(L_i - L_j)$ . Since new customers differ in their values of  $\tau$ , market demand for the product will be downward sloping and the compatibility regime will affect the total number of new customers. Following Katz and Shapiro [1985], we assume  $\tau$  is uniformly distributed over  $(-\infty, 1]$ , with density equal to 1 over this interval.<sup>9</sup> Instead of a literally infinite potential demand, we view this as capturing the realistic feature that the pool of potential customers is never exhausted.<sup>10</sup> The marginal customer,  $\bar{\tau}$ , obtains

<sup>7</sup> Thus, consumers differ only in their stand-alone valuations. Bental and Spiegel [1995] and Economides and Flyer [1997] instead assume that consumers differ in their willingness to pay for the network effect—a consumer's type enters multiplicatively with the network term. This introduces interesting new possibilities: even with *ex ante* symmetric firms, total incompatibility is an equilibrium and the resulting firm sizes and profits are asymmetric—as in standard models of vertical differentiation with heterogeneous consumers.

<sup>8</sup> The assumption that a consumer's benefit rises with  $L_i$  most naturally captures direct network effects, but it can capture in a reduced-form manner also indirect network effects if a larger number of customers who use compatible products elicits a better supply (lower price or greater variety) of complements. In communications industries (the epitome of direct network effects), a compatibility level of 1 means that 'off-net' communications are as good as 'on-net'; in a hardware/software setting (exhibiting indirect network effects), it means that complementary products designed for firm  $i$ 's platform can interoperate equally well with another firm's platform.

<sup>9</sup> Exact agreement with Katz and Shapiro's setting would model parameter  $\tau$  as uniformly distributed over  $(-\infty, A]$ , for  $A > 0$ . Our current results would extend after slight reinterpretation. For example,  $\tau$  would be replaced by  $\tau/A$  and  $c$  by  $c/A$ . We set  $A = 1$  without loss of generality. *Our assumption of a uniform distribution should not be mistaken for a uniform probability distribution, which requires a bounded support.*

<sup>10</sup> Katz and Shapiro [1985, fn. 2] introduce this 'unbounded below' assumption specifically to eliminate problems with solutions in which all potential consumers are served. By contrast, CRT assume that the mass of potential customers is finite (and uniformly distributed over  $[0, 1]$ ), which leads to corner solutions for some parameter values. Corner solutions raise issues of

zero net surplus and all consumers with  $\tau \in [\bar{\tau}, 1]$  will purchase, so the number of new customers is  $1 - \bar{\tau}$ . Letting  $q_j$  denote the number of new customers added by firm  $j$ , equilibrium requires  $\sum_{j=1}^{n+1} q_j = 1 - \bar{\tau}$ . Using  $\bar{\tau} = 1 - \sum_{j=1}^{n+1} q_j$  in (1) shows the market-clearing price for each firm  $i$  that adds customers in a consistent-expectations equilibrium satisfies

$$(2) \quad p_i = 1 + vL_i - \sum_{j=1}^{n+1} q_j.$$

All firms have constant marginal cost  $c$  of serving additional customers. We analyze equilibria in which the expectations of consumers and firms about all  $q_i$ , and hence network sizes  $L_i, i = 1, \dots, n+1$ , are confirmed. Firms compete for new customers in a Cournot fashion: in the second period, each firm chooses the number of customers it wishes to add, and firms' prices adjust to clear the market, given the correctly expected number of new customers for each firm. This competition takes place after the compatibility regime has been chosen, as discussed next.

#### II(ii). *Full Compatibility vs. Autarky by Firm 1*

The total installed base in the industry is  $\beta > 0$ . Firm 1 has the largest base, of size  $\beta_1$ , with market share  $m_1 \equiv \beta_1/\beta$ . Throughout the paper, 'market share' refers to the installed-base share, and firm 1 is the 'largest firm' in this sense. There are  $n$  smaller rivals.

*Compatibility Assumptions, Network Sizes, and Inverse Demands* We make the following permanent assumptions. First, there are no payments between firms for compatibility. Second, the compatibility quality between two firms is  $\theta \in (0, 1)$ ;  $\theta = 1$  and cannot be attained unilaterally (no converters); and both qualities entail equal cost, normalized to zero.<sup>11</sup> Until Section VI, we also assume (A1) uniform compatibility policies—any firm must offer the same compatibility level, 0 or 1, to all other firms; and (A2) each smaller rival offers compatibility. In Section VI we describe an environment in which (A1) and (A2) arise endogenously, and also discuss departures from these assumptions.

multiple equilibria (Malueg and Schwartz [2002, fn. 8]). Malueg and Schwartz retain CRT's assumption, but take the perspective that the plausible market outcome is one in which not all potential customers will subscribe; constraining the total number of new subscribers to be less than the potential pool yields a restriction on plausible combinations of the model's parameters.

<sup>11</sup> CRT [2000] show that when quality level  $\theta$  is continuous but all qualities entail equal cost, then in equilibrium firms in fact choose either 0 or 1, thereby offering a separate justification for the 0–1 assumption.

Under (A1) and (A2), firm 1 chooses between two compatibility regimes: if firm 1 chooses compatibility, we have *full compatibility*, while if it denies compatibility we have *autarky by firm 1* facing the compatible rivals. Given that rivals are compatible, their individual installed-base shares will not matter for competition, only their total share  $1 - m_1$  is relevant.

Competition for new customers occurs after the compatibility regime is known. Under autarky by firm 1, each rival engages in intra-network competition with the other rivals and inter-network competition versus firm 1; with full compatibility, there is only intra-network competition. The profit of any firm  $i$  from new customers is  $\pi_i = (p_i - c)q_i$ , where firm 1's choice of  $\theta$  affects the size,  $L_i$ , of the network accessible to each firm and, hence, the inverse demand function  $p_i$ . The number  $L_i$  includes the customers of firm  $i$  (installed-base and new ones) as well as of other firms that are compatible with  $i$ . Because all smaller firms are compatible, the network of any firm  $i$ ,  $i \neq 1$ , includes all customers of the smaller firms; it also includes firm 1's customers if and only if firm 1 chooses compatibility ( $\theta = 1$ ). Thus,

$$(3) \quad L_i = (\beta - \beta_1) + \sum_{j=2}^{n+1} q_j + \theta(\beta_1 + q_1), i = 2, \dots, n + 1.$$

Similarly, the size of firm 1's network is given by

$$(4) \quad L_1 = (\beta_1 + q_1) + \theta \left( \beta - \beta_1 + \sum_{j=2}^{n+1} q_j \right).$$

Using (4) in (2), given correct consumer expectations about network sizes, we find the inverse demand facing firm 1 is

$$(5) \quad \begin{aligned} p_1 &= 1 + vL_1 - q_1 - \sum_{j=2}^{n+1} q_j \\ &= 1 + v(\beta_1 + \theta(\beta - \beta_1)) - (1 - v)q_1 - (1 - \theta v) \sum_{j=2}^{n+1} q_j; \end{aligned}$$

similarly, using (3) in (2), we find the inverse demand facing any smaller firm  $i$  is

$$(6) \quad \begin{aligned} p_i &= 1 + vL_i - q_i - \sum_{j \neq i} q_j \\ &= 1 + v(\beta - \beta_1 + \theta\beta_1) - (1 - v) \left( q_i + \sum_{j \neq 1, i} q_j \right) - (1 - \theta v)q_1, \quad i = 2, \dots, n + 1. \end{aligned}$$



We restrict  $v < 1$  to guarantee that demand is downward-sloping (see (5) and (6)).<sup>12</sup> Also, we restrict  $c < 1$  to ensure that some new customers are added under full compatibility, no matter how small are  $v$  or  $\beta$ : among the potential new customers, the highest willingness to pay under full compatibility is at least  $1 + v\beta$ , which is sure to exceed marginal cost if and only if  $c \leq 1$  (we exclude  $c = 1$  for purely technical reasons).

*Equilibrium under Full Compatibility and Firm 1's Autarky Tradeoffs* Under full compatibility, all firms' products are perfect substitutes. Using the inverse demands given in (5) and (6) with  $\theta = 1$ , firm  $i$ 's profit can be expressed as

$$(7) \quad \pi_i = (p_i - c)q_i = \left( 1 + v\beta - (1 - v) \left( q_i + \sum_{j \neq i} q_j \right) - c \right) q_i,$$

$i = 1, 2, \dots, n + 1$ . At the Cournot equilibrium, each firm  $i$  maximizes its profit in (7), taking as given the correctly expected outputs of its rivals. Given  $v < 1$  and  $c < 1$ , the Cournot equilibrium under full compatibility is unique, with each firm's adding the identical number of customers  $q^a$  and enjoying the same network size  $L$ :

$$(8) \quad q^a = \frac{(1 - c) + v\beta}{(n + 2)(1 - v)} \text{ and } L = \beta + (n + 1)q^a,$$

where the superscript  $a$  indicates that firm 1 accepts compatibility (below we use a superscript  $d$  to indicate that firm 1 declines compatibility). Observe that the equilibrium number of new customers per firm decreases with marginal cost  $c$  and with the number of rivals  $n$ , but the total number of new customers,  $(n + 1)q^a$ , rises with  $n$ ; the number of new customers increases with the installed base  $\beta$  and valuation parameter  $v$ , since both enhance network attractiveness.

Under autarky by firm 1, the equilibrium network qualities for firm 1 and any rival  $j$  are

$$L_1 = \beta_1 + q_1^d \text{ and } L_j = \beta - \beta_1 + nq_j^d,$$

where  $q_1^d$  and  $q_j^d$  are the equilibrium new outputs. Before deriving these outputs in Section III, we provide a decomposition that offers some insights

<sup>12</sup> The role of  $v < 1$  can be understood as follows. Consider perfect compatibility. Suppose at price  $p$  the marginal consumer has stand-alone value equal to  $\tau$ . If  $\Delta\tau$  more consumers are to be added, then the stand-alone value of the new marginal consumer must be lower by  $\Delta\tau$  (since  $\tau$  is uniformly distributed over  $(-\infty, 1]$ ). The quality of the expanded network, however, rises by  $v\Delta\tau$ , so the overall value to the marginal consumer (which determines the market price) would fall by just  $(1 - v)\Delta\tau$ . Thus, in order for marginal willingness-to-pay to fall as the number of customers increases, it is necessary that  $v < 1$ .

into firm 1’s tradeoffs if it moves from compatibility to autarky. Firm 1’s inverse demand, given that rivals add  $Q_R$  customers, is

$$p_1 = 1 + vL_1 - q_1 - Q_R.$$

Holding rivals’ total output fixed at its equilibrium level under compatibility or autarky,  $Q_R^a$  or  $Q_R^d$ , the difference in the price firm 1 can obtain in the two regimes for any common output  $q_1$  is

$$(9) \quad p_1^d|_{Q_R^d} - p_1^a|_{Q_R^a} = \underbrace{(Q_R^a - Q_R^d)}_{\text{Rivals' Contraction (RC)}} (1 - v) - \underbrace{[\beta(1 - m_1) + Q_R^d]}_{\text{Lost Links (LL)}} v.$$

Although firm 1’s equilibrium outputs will generally differ under compatibility and autarky, firm 1’s equilibrium profit is higher under autarky if and only if (9) is positive. This follows because in each equilibrium, firm 1 chooses its best-response output given the residual inverse demand implied by rivals’ total output in that same equilibrium. If the difference in (9) is positive, firm 1 faces higher inverse demand under autarky, and hence earns higher profit.<sup>13</sup>

The sign of (9) depends on two effects induced by moving from compatibility to autarky. *Lost Links* are the number of customers served by rivals collectively in the autarky equilibrium; this loss devalues firm 1’s product in portion to the network-effects parameter  $v$ . While  $Q_R^d$  can be zero (as occurs if autarky yields tipping to firm 1, instead of an interior equilibrium), firm 1 loses access at least to rivals’ installed base  $\beta(1 - m_1)$ ; the Lost Links effect therefore always harms firm 1.<sup>14</sup> Thus, autarky can benefit firm 1 only if it induces positive *Rivals’ Contraction*,  $Q_R^a - Q_R^d > 0$ .<sup>15</sup>

<sup>13</sup> Firm 1’s equilibrium profit under autarky is the maximum with respect to  $q_1$  of  $(p_1^d|_{Q_R^d}(q_1) - c)q_1$  and under compatibility it is the maximum of  $(p_1^a|_{Q_R^a}(q_1) - c)q_1$ . Autarky profit is higher if  $p_1^d|_{Q_R^d}(q_1) > p_1^a|_{Q_R^a}(q_1)$  for all  $q_1$ , while compatibility profit is higher if the reverse inequality holds.

<sup>14</sup> Indeed, this force makes it possible for autarky to reduce firm 1’s profit relative to compatibility even if the autarky equilibrium would involve the market’s tipping to firm 1 (see Section IV(i)).

<sup>15</sup> This benefits firm 1 given  $v < 1$ , the same condition that ensured downward-sloping demand under network effects. When autarky by firm 1 reduces rivals’ output, this has opposing effects on the price firm 1 can charge if it wishes to add  $q_1$  customers: (a) the marginal customer has higher basic valuation  $\tau$  since (by hypothesis) rivals draw fewer customers from the given pool, but (b) any customer of firm 1 gets lower network benefits. If  $v < 1$ , the standard competition effect, (a), dominates the network effect, (b), so firm 1 gains from rivals’ contraction. In other settings, Conner and Rumelt [1991] and Takeyama [1994] show that strong network effects enable an intellectual-property owner to benefit from unauthorized copying of its product, provided the lost customers remain compatible with its own. (In Conner and Rumelt, this arises because some copiers would drop out altogether if forced to pay; in Takeyama, copying lets the firm attain a maximal network size at a higher price to those (high-value) users who still purchase than if it had to induce the lower-value users to join the network by buying rather than copying.)

However,  $Q_R^a - Q_R^d < 0$  is also possible. While autarky by firm 1 reduces all firms' absolute quality, it can improve rivals' quality *relative* to 1—even if 1's installed base is larger ( $m_1 > 1/2$ ). The rivals' advantage stems from the *intra-network competition* effect: a network of two or more compatible rivals is expected to act more aggressively in *adding* customers due to competition among them. This effect can outweigh firm 1's installed-base advantage. Consequently, if firm 1 chooses autarky its share of *new* customers may drop below what it would enjoy under compatibility,  $1/(n+1)$ , and even to zero if the market tips from firm 1 to the rivals (see Section III.(i)). We shall return to decomposition (9) when analyzing how various parameters affect the profitability of autarky (Section IV.(ii)).<sup>16</sup>

To determine which regime firm 1 prefers, we first identify the possible outcomes—interior or tipping equilibria—conditional on firm 1 choosing autarky. Then, in Section IV, we compare firm 1's profit in these outcomes with its profit under full compatibility.

### III. POSSIBLE OUTCOMES IF THE LARGEST FIRM CHOOSES AUTARKY

If firm 1 denies compatibility while its rivals are compatible, then, depending on parameter values, the *unique* equilibria is either: A) *interior*—all firms obtain new customers; or B) *tipping to 1*—only firm 1 obtains new customers; or C) *tipping from 1*—only firm 1 obtains *no* new customers; for different parameter values, there are D) *multiple equilibria*—depending on consumers' expectations; the realized outcome can be A, B or C. Figure 1 illustrates these possibilities in  $(m_1, v)$  space for given values of the other parameters  $c, \beta$ , and  $n$ , depending on whether firm 1's installed-base share  $m_1$  is larger or smaller than certain thresholds  $\underline{M}_1$  and  $\bar{M}_1$ :

- region A:  $m_1 < \underline{M}_1$  and  $m_1 > \bar{M}_1$ —*interior equilibrium*;
- region B:  $m_1 \geq \underline{M}_1$  and  $m_1 > \bar{M}_1$ —*tipping to 1*;
- region C:  $m_1 < \underline{M}_1$  and  $m_1 \leq \bar{M}_1$ —*tipping from 1*;
- region D:  $m_1 \geq \underline{M}_1$  and  $m_1 \leq \bar{M}_1$ —*multiple equilibria*.

The next section derives the thresholds  $\underline{M}_1$  and  $\bar{M}_1$  (as functions of various parameters) and establishes this taxonomy of possible equilibrium configurations.

<sup>16</sup> There is an imperfect analogy between this network-effects setting and a standard raising-rivals'-costs model (e.g., Salop and Scheffman [1987]), where a dominant firm can gain from an action that raises its rivals' costs even if its own cost also rises. Here, by denying compatibility, firm 1 lowers all firms' qualities but can benefit if rivals' quality falls by more. The analogy would be exact *if* new consumers only valued compatibility with installed-base consumers, as equilibrium qualities then would depend only on the *exogenous* installed bases. But since new consumers also value access to other new ones, relative qualities under autarky by firm 1 depend on the network choices of new consumers (firm 1's or the rivals')—choices that depend on expectations about other consumers' choices. Thus, autarky by firm 1 can yield multiple equilibria, rendering the network-effects problem more complex.

III(i). *Tipping and Interior Equilibrium*

*Tipping To Firm 1* A tipping equilibrium to firm 1 under autarky and Cournot competition is described as follows. Suppose each potential new customer expects every new customer to choose firm 1. Given such expectations, firm 1 adds its monopoly number of new customers; taking as given that firm 1 will add this number (and charge the market-clearing price), no rival can profitably attract any new customers.<sup>17</sup> We now derive the condition for such a tipping equilibrium to exist.

If  $q_2 = \dots = q_{n+1} = 0$ , then firm 1's output (i.e., number of new customers) is

$$(10) \quad q_1^{Tip} = \frac{(1-c) + \beta_1 v}{2(1-v)},$$

which is simply firm 1's monopoly output—its best response to zero output by rivals. Given  $q_1^{Tip}$  and zero output by the other rivals, any rival firm  $i$  will choose zero output, if  $p_i \leq c$  for any  $q_i > 0$ , where  $p_i$  is firm  $i$ 's inverse demand function given by (6). At these candidate equilibrium outputs,  $q_1^{Tip}$  and  $q_2 = \dots = q_{n+1} = 0$ , we indeed have  $p_i \leq c$  if and only if

$$(11) \quad 1 + (\beta - \beta_1)v - \left( \frac{1-c + \beta_1 v}{2(1-v)} \right) \leq c.$$

Letting  $\beta_1 = m_1 \beta$ , we transform (11) into a condition expressing the *minimum* market share of firm 1 for which, under autarky, there exists an equilibrium with tipping to firm 1:

$$(12) \quad m_1 \geq \underline{M}_1(c, v, \beta) \equiv \frac{2(1-v)}{(3-2v)} + \frac{(1-2v)}{(3-2v)v\beta}(1-c). \quad (\text{Tipping to 1})$$

Thus, tipping to 1 can be an equilibrium if and only if 1's market share is sufficiently large—given  $v$ ,  $m_1$  lies to the right of the  $\underline{M}_1$  in Figure 1. Lemma 1 establishes the shape of this curve. The proofs of Lemma 1 and subsequent results are given in the Appendix.

*Lemma 1.* Fix  $c$ ,  $\beta$ , and  $n$ . In  $(m_1, v)$  space, the graph of  $\underline{M}_1$  is strictly decreasing in  $v$  and passes through the point  $(m_1, v) = (1/2, 1/2)$ . This curve is independent of  $n$ .

The curve  $\underline{M}_1$  is negatively sloped because tipping to 1 is facilitated by stronger network effects and by a larger installed-base share for firm 1. Thus,

<sup>17</sup> Because of the network effects, entry by a rival on a small scale would deliver a service of low quality, which would have to be compensated by a prohibitively lower price; entry on a large scale would require a large price cut to achieve the requisite market expansion, again driving price below marginal cost.

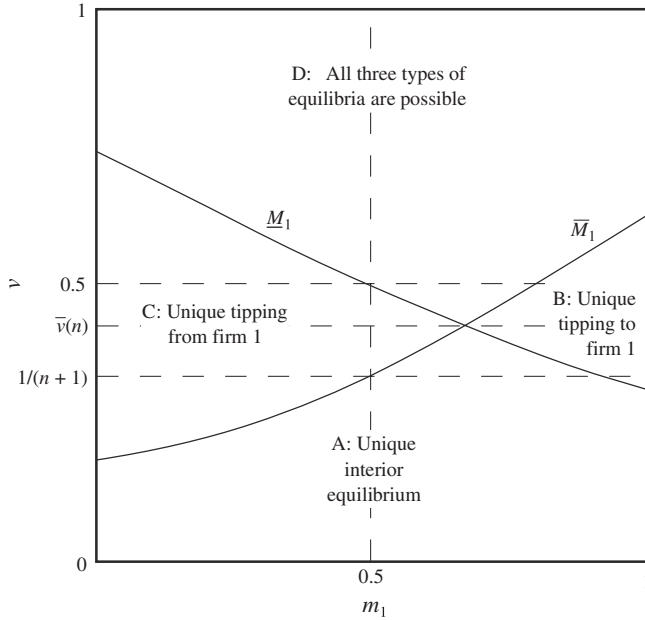


Figure 1  
Possible Equilibria if Firm 1 Chooses Autarky:  $c = 0.2, \beta = 1, n = 2$

increasing  $v$  reduces the minimal  $m_1$  needed for tipping to 1. The curve  $\underline{M}_1$  is unaffected by the number of rivals  $n$  because tipping to 1 arises if marginal cost  $c$  is weakly above rivals' inverse demand at zero output for them and firm 1's monopoly output ( $q_1^{Tip}$ , see (10)), which is independent of  $n$ .

*Tipping From Firm 1* Next consider tipping *from* firm 1 to the rivals' network. Suppose each potential new customer expects that no new customer will choose firm 1. Given such expectations, each of the  $n$  rivals adds the symmetric Cournot equilibrium number of new customers,

$$(13) \quad q_i^{Tip} = \frac{(1 - c) + (\beta - \beta_1)v}{(n + 1)(1 - v)}, \quad i = 2, 3, \dots, n + 1.$$

When tipping is from 1, it is convenient to denote rivals' aggregate output by

$$(14) \quad Q_R^{Tip} \equiv \sum_{i=2}^{n+1} q_i^{Tip} = \frac{n(1 - c + (\beta - \beta_1)v)}{(n + 1)(1 - v)}.$$

The outputs  $q_1 = 0$  and  $q_i = q_i^{Tip}$  indeed form a tipping equilibrium *from* firm 1 if at these outputs  $p_1 \leq c$ , where  $p_1$  is firm 1's inverse demand function

given by (5):

$$(15) \quad 1 + \beta_1 v - \frac{n(1 - c + (\beta - \beta_1)v)}{(n + 1)(1 - v)} \leq c.$$

Again letting  $\beta_1 = m_1\beta$ , we now transform (15) into a condition expressing the *maximum* market share of firm 1 for which, under autarky, there exists a tipping equilibrium from firm 1:

$$(16) \quad m_1 \leq \bar{M}_1(c, v, \beta, n) \equiv \frac{n}{n + (n + 1)(1 - v)} + \frac{((n + 1)v - 1)}{(n + (n + 1)(1 - v))v\beta} (1 - c). \quad (\text{Tipping from 1})$$

Thus, tipping from 1 can be an equilibrium if and only if 1’s market share is not too large—given  $v$ ,  $m_1$  lies to the left of the curve  $\bar{M}_1$  in Figure 1. Lemma 2 derives the shape of this curve.

*Lemma 2.* Fix  $c$ ,  $\beta$ , and  $n$ . In  $(m_1, v)$  space, the graph of  $\bar{M}_1$  is strictly increasing in  $v$  and passes through the point  $(m_1, v) = (1/2, 1/(n + 1))$ . This curve shifts right and down as  $n$  increases.

The curve  $\bar{M}_1$  is positively sloped because tipping from 1 is facilitated by stronger network effects and by a *lower* installed-base share of firm 1. Unlike  $\underline{M}_1$ ,  $\bar{M}_1$  *does* depend on the number of rivals: for a given installed-base share of firm 1, an increase in  $n$  expands the rivals’ future Cournot output, which depresses firm 1’s residual inverse demand. Consequently, increasing  $n$  makes tipping from 1 possible for higher values of  $m_1$ .

*Interior Equilibrium* For some parameter values, autarky by firm 1 would yield a unique interior equilibrium. In this case, firm 1 chooses its number of new customers,  $q_1$ , to maximize its profit

$$(17) \quad \pi_1 = (p_1 - c)q_1 = \left( 1 + v\beta_1 - (1 - v)q_1 - \sum_{j=2}^{n+1} q_j - c \right) q_1.$$

Among the rivals, any firm  $i$  chooses its number of new customers,  $q_i$ , to maximize

$$(18) \quad \begin{aligned} \pi_i &= (p_i - c)q_i \\ &= \left( 1 + v(\beta - \beta_1) - (1 - v) \left( q_i + \sum_{j \neq 1, i} q_j \right) - q_1 - c \right) q_i. \end{aligned}$$

If the Cournot equilibrium has all firms adding new subscribers, then the equilibrium outputs are

$$(19) \quad q_1^d = \frac{(1 - c)[1 - (1 + n)v] + [n + (n + 1)(1 - v)] v\beta_1 - nv\beta}{2(n + 1)(1 - v)^2 - n}$$

for firm 1, and for the rivals,

$$(20) \quad q_i^d = \frac{(1 - c)(1 - 2v) - (3 - 2v)v\beta_1 + 2(1 - v)v\beta}{2(n + 1)(1 - v)^2 - n}$$

$i = 2, 3, \dots, n + 1.$

Firm 1’s rivals all have the same number of new customers, regardless of their individual installed bases, because the rivals are compatible and hence offer identical services.

Equations (19) and (20) provide the equilibrium outputs if these formulas yield well-defined and strictly positive outputs, which requires both numerators and denominators to be strictly positive or strictly negative. The denominators in (19) and (20) equal zero at

$$(21) \quad \bar{v}(n) \equiv 1 - \sqrt{\frac{n}{2(n + 1)}},$$

are strictly positive for  $v < \bar{v}(n)$ , and are strictly negative for  $v > \bar{v}(n)$ .<sup>18</sup> Therefore, the interior equilibrium exists in two cases:

Case 1.  $v < \bar{v}(n)$  and both 1(i) and 1(ii) hold:

1(i) Numerator of (19)  $> 0$

1(ii) Numerator of (20)  $> 0$ ;

Case 2.  $v > \bar{v}(n)$  and both 2(i) and 2(ii) hold:

2(i) Numerator of (19)  $< 0$

2(ii) Numerator of (20)  $< 0$ .

The conditions of Cases 1 and 2 can be converted to conditions on firm 1’s market share,  $m_1$ , by setting  $\beta_1 = m_1\beta$  in the numerators of (19) and (20):

Case 1.  $v < \bar{v}(n)$

$$(22) \quad 1(i) \ m_1 > \frac{n}{n + (n + 1)(1 - v)} + \frac{((n + 1)v - 1)}{(n + (n + 1)(1 - v))v\beta} (1 - c) = \bar{M}_1(c, v, \beta, n)$$

$$(23) \quad 1(ii) \ m_1 < \frac{2(1 - v)}{(3 - 2v)} + \frac{(1 - 2v)}{(3 - 2v)v\beta} (1 - c) = \underline{M}_1(c, v, \beta);$$

<sup>18</sup> The critical  $\bar{v}$  decreases in  $n$ , with  $\bar{v}(1) = 1/2$  and  $\lim_{n \rightarrow \infty} \bar{v}(n) = 1 - \sqrt{1/2} \approx 0.293$ .

Case 2.  $v > \bar{v}(n)$

$$(24) \quad 2(i) \ m_1 < \bar{M}_1(c, v, \beta, n)$$

$$(25) \quad 2(ii) \ m_1 > \underline{M}_1(c, v, \beta).$$

The equalities in (22) and (23) follow from the definitions in (16) and (12), respectively. Recalling  $\bar{v}(n)$  from (21), Lemma 3 states that  $\underline{M}_1$  lies to the right of  $\bar{M}_1$  for  $v < \bar{v}$  and to the left for  $v > \bar{v}$ , as shown in Figure 1.

*Lemma 3.* Fix  $c$ ,  $\beta$ , and  $n$ . Then  $\underline{M}_1 \cong \bar{M}_1$  if and only if  $v \cong \bar{v}(n)$ .

The analysis of Section III(i) has established that the possible equilibrium configurations are those shown in Figure 1.<sup>19</sup>

### III(ii). *Intra-Network Competition and Tipping*

We shall discuss how the model's parameters affect the scope for various equilibria. Let  $x$  designate any subset of the model parameters  $c$ ,  $v$ ,  $\beta$ ,  $n$ , or  $m_1$ ; given  $x = x_0$  we say that the *scope for E* is the set of vectors of parameters other than  $x$  for which the event  $E$  holds when  $x = x_0$ . For example, if  $n = n'$  then the scope for equilibrium tipping to 1 is the set of vectors  $(c, v, \beta, m_1)$  for which  $(c, v, \beta, n')$  supports such an equilibrium. We say that a change in a parameter *increases* the scope for  $E$  if the corresponding set of parameters expands in the sense of set inclusion. Finally, 'the equilibrium can be of type X' means there exist parameter values that support an equilibrium of type X (e.g., tipping to 1).

As expected, under autarky stronger network effects increase the scope for tipping.<sup>20</sup> Thus, in Figure 1 raising  $v$  can move the environment from a region of no tipping into one with unique tipping (i.e., from A into B or C), or

<sup>19</sup> To understand this taxonomy, recall that the interior equilibrium exists if one of the following cases holds: Case 1:  $v < \bar{v}$ ,  $m_1 > \bar{M}_1$ , and  $m_1 < \underline{M}_1$  (see (22) and (23)); or Case 2:  $v > \bar{v}$ ,  $m_1 < \bar{M}_1$ , and  $m_1 > \underline{M}_1$  (see (24) and (25)). Consequently, an interior equilibrium exists only in region A (Case 1) and in D (if its inequalities hold strictly—Case 2). In A, the inequalities rule out both tipping equilibria; hence the unique equilibrium is interior. In D the inequalities admit both tipping equilibria. Finally, in B, the inequalities admit only tipping to firm 1, and in C the inequalities admit only tipping from 1 (see (12) and (16) above).

<sup>20</sup> Although expected, this result is not straightforward, at least in this model. Tipping to 1 requires  $1 + v(\beta - \beta_1) - q_1^{tip} \leq c$  (see (10) and (11)). Increasing  $v$  has opposing effects on the left-hand side, rivals' inverse demand: (i) positive, since the value of connectivity to the rivals' installed base, the term  $v(\beta - \beta_1)$ , increases, and (ii) negative, since firm 1's monopoly output  $q_1^{tip}$  also increases (because raising  $v$  makes firm 1's demand curve more elastic), leaving the rivals a pool of customers whose willingness to pay (their type,  $\tau$ ) is lower. The proof of Lemma 1 shows that effect (ii) dominates, so raising  $v$  encourages tipping to 1 (it can occur at a lower installed-base share for 1). A similar analysis applies to tipping from 1 (see equations (13), (15), and the proof of Lemma 2).



from a region with unique tipping into the region that admits both tipping equilibria (from B or C into D).

The next result shows how firm 1's installed-base share,  $m_1$ , and the number of rivals,  $n$ , affect whether the equilibrium conditional on autarky is unique and involves tipping.

*Proposition 1 (Unique tipping conditional on autarky).* Suppose firm 1 chooses autarky.

- (i) If  $m_1 \leq 1/2$ , tipping from firm 1 can be the unique equilibrium but tipping to 1 cannot.
- (ii) If  $m_1 > 1/2$ , tipping from 1 can be the unique equilibrium if and only if firm 1 faces at least two rivals.
- (iii) Raising the number of rivals (a) increases the scope for unique tipping from firm 1 and (b) decreases the scope for unique tipping to firm 1.

Part (iii) is illustrated in Figure 2, which is similar to Figure 1 but includes an additional curve  $\bar{M}_1$  corresponding to a larger number of rivals. Fix  $c, \beta$ , and  $n'' > n' \geq 2$ . Because  $n' \geq 2$ , the left-side vertical intercept for each curve  $\bar{M}_1$  lies strictly below that for  $\underline{M}_1$ . Tipping to 1 occurs above  $\underline{M}_1$ ; tipping from 1 occurs above  $\bar{M}_1$ . Because  $\underline{M}_1$  is independent of  $n$ , so is the region of tipping to 1. However, the curve  $\bar{M}_1$  shifts down as  $n$  increases. Thus, the region of

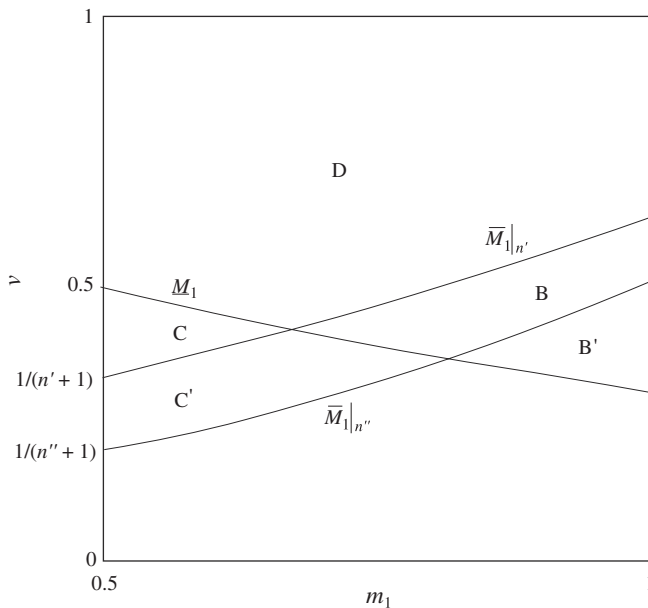


Figure 2  
Number of Rivals and Tipping Equilibria:  $c = 0.2, \beta = 1, n' = 2, n'' = 4$

tipping from 1 strictly expands (from  $C \cup D$  to  $C \cup D \cup B \cup C'$ ); correspondingly, the region of *unique* tipping from 1 also strictly expands (from  $C$  to  $C \cup C'$ ), while the region of unique tipping to 1 strictly shrinks (from  $B \cup B'$  to  $B'$ ). We now discuss the intuition behind Proposition 1, especially parts (ii) and (iii).

Given the central role of consumer expectations in environments with network effects, it is not surprising that autarky by firm 1 admits multiple equilibria (see Katz and Shapiro [1985]), including tipping from firm 1 even if it has more than half of the installed base. Less obvious is why tipping from 1 can then be the *unique* equilibrium. The reason is the *intra-network competition effect*: for the same installed base (hence, the same initial quality), a network of  $n \geq 2$  compatible but competing rivals would add more new customers than would firm 1 as a monopolist. Intra-network competition makes it possible that, despite rivals' installed-base disadvantage, their network would add more customers if consumers expected it to be the sole network than would firm 1 as the sole network. If this differential expansion is sufficiently large, then rivals' tipping output may depress firm 1's residual demand below marginal cost, but not vice versa—explaining Proposition 1(ii). Part (iii) follows because the strength on intra-network competition increases with the number of rivals. Part (i) follows because firm 1's advantage in achieving a tipping equilibrium to it rather than to rivals can only come from a larger installed base.

To see these effects more clearly, let  $R$  denote the network of  $n$  compatible rivals,  $Q_R$  its total output,  $\beta_R (\equiv \beta - \beta_1)$  its installed base, and  $p_R$  the common inverse demand facing any rival firm  $i$ . Under autarky by firm 1, the inverse demands facing networks 1 and  $R$  are derived using (5) and (6) with  $\theta = 0$ :  $p_1 = 1 + v\beta_1 - Q_R - (1 - v)q_1$  and  $p_R = 1 + v\beta_R - q_1 - (1 - v)Q_R$ . In a tipping equilibrium to network  $j$  ( $j = 1$  or  $R$ ), the other network  $i$  cannot profitably attract any new customers, given  $i$ 's installed base and  $j$ 's number of new customers. Denote by  $Q(\beta', n')$  the total Cournot output of a network that has  $n'$  compatible firms with total installed base  $\beta'$  and is the only network in the market.<sup>21</sup> Recalling (11) and (15), tipping to firm 1 requires

$$(26) \quad c \geq p_R \Big|_{\substack{q_1 = q_1^{Tip} \\ Q_R = 0}} = 1 + v\beta_R - Q(\beta_1, 1), \text{ (Tipping to 1)}$$

and tipping from 1 requires

$$(27) \quad c \geq p_1 \Big|_{q_1 = 0} = 1 + v\beta_1 - Q(\beta_R, n). \text{ (Tipping from 1)}$$

*Unique* tipping from 1 therefore requires (27) to be satisfied while (26) is not:

$$(28) \quad 1 + v\beta_1 - Q(\beta_R, n) \leq c < 1 + v\beta_R - Q(\beta_1, 1).$$

<sup>21</sup> From (10) and (14) we see that  $Q(\beta_1, 1) = q_1^{Tip}$  and  $Q(\beta_R, n) = Q_R^{Tip}$ .

The interval for  $c$  described in (28) is non-empty if and only if

$$(29) \quad v(\beta_1 - \beta_R) < Q(\beta_R, n) - Q(\beta_1, 1).$$

The left term of (29) is positive and reflects firm 1's quality advantage from its larger installed base. The right term reflects rivals' *potential* advantage in raising quality by *adding* customers. Its sign is generally ambiguous, because a network's total output  $Q(\beta', n')$  is increasing in both arguments. When  $\beta_1 > \beta_R$  and  $n \geq 2$ , there are two opposing effects. The rivals' network has a smaller installed base, which reduces its new output; however, intra-network competition increases its output. As the installed base  $\beta$  falls to zero so does the left term in (29), while the right term remains positive given  $n \geq 2$ . Thus, if  $\beta$  is sufficiently small, condition (29) can be met for any value of  $m_1$ , which explains why unique tipping from 1 is then possible—the 'if' part of Proposition 1(ii). The 'only if' part follows because intra-network competition requires at least two rivals. Since the intra-network competition effect is stronger with more rivals, the scope for unique tipping from 1 expands with  $n$  (Proposition 1(iii)).

It is worth comparing these tipping results with the analysis by Katz and Shapiro [1986] of competition between two incompatible technologies/products, one of which is 'sponsored' (is proprietary to a single firm) and the other is open to all firms. In their model, consumers differ only in when they purchase, period 1 or 2, and their value depends only on final (second-period) network sizes. The competing products are identical except for their network sizes. Unlike in our model, competition is in prices, so in the last period only one technology is chosen—an interior equilibrium is not possible. For most of the analysis, consumers have unit demands, so all will fully purchase in either equilibrium. The sponsored technology then has a strategic advantage: only it is willing to price below marginal cost in the first period to build up an installed-base advantage so as to attract second-period consumers, because only it can hope to recoup penetration-pricing losses by pricing above cost later. (Bertrand competition between suppliers of the other product always keeps their prices at cost.) Importantly, above-cost final-period pricing does not shrink the sponsored technology's network size, because of the identical unit demands. Katz and Shapiro (p. 839) caution, however, that if demands are *elastic*, then above-cost pricing by the sponsored technology will reduce its expected network size and this effect can outweigh the penetration-pricing advantage: 'One can show by example that, in the first period, the sponsored technology may be at a disadvantage relative to the unsponsored one because [only] the latter can credibly commit to marginal cost pricing.' Thus, our findings are consistent with Katz and Shapiro's for the case of elastic demand.

Economides [1996b] also highlights that intra-network competition can help convince consumers that future prices will be lower and network size

larger. If network effects are sufficiently strong, a monopolist can profit by licensing competitors even for free, because—absent commitments to limit a monopolist's future behavior—'. . . higher expectations of sales [yielding higher willingness to pay due to the larger network effect] can only be fulfilled at equilibrium in a more competitive market with a larger number of participants.'<sup>22</sup>

Finally, the strategic role of competition as a commitment device, and its concomitant value in influencing expectations, has been noted also outside of network settings. Schwartz and Thompson [1986] show that an incumbent may gain from establishing competing divisions for purposes of deterring entry by other firms. Farrell and Gallini [1988] show that an innovator may prefer to have more than one licensee despite the profit destruction caused by licensees' competition because, given incomplete contracts, this competition helps protect consumers against future *ex post* opportunism and thus induces them to undertake specialized investments complementary to the innovator's product.

#### IV. PROFITABILITY OF AUTARKY FOR THE LARGEST FIRM

We shall say that autarky *clearly profitable* or *clearly unprofitable* for firm 1 at  $(c, v, \beta, n, m_1)$  if autarky yields a unique equilibrium and firm 1's profit there is, respectively, strictly higher or strictly lower than its profit under compatibility. In the other parameter regions, the profitability of autarky is ambiguous, depending on which of the multiple equilibria emerges as the outcome (which will depend on consumers' expectations). Observe that in any equilibrium—tipping or interior—firm 1's profit is  $\pi_1 = (1 - v)(q_1^*)^2$ , where  $q_1^*$  denotes firm 1's output in that equilibrium.<sup>23</sup> Therefore, in the first stage firm 1 will select (by its choice of  $\theta = 0$  or 1) the compatibility regime that gives it greater equilibrium output.

##### IV(i). *Conditions for Profitable Autarky*

First consider parameter values for which the unique equilibrium under autarky is *interior*, with outputs given by (19) and (20). From Section III it follows that a necessary condition for autarky to yield this unique

<sup>22</sup> Economides [1996b, p. 213]. He credits Katz and Shapiro [1985, p. 431] with the original insight that inviting entry can raise profit: 'a monopolist will exploit his position with high prices and consumers know this. Thus, consumers expect a smaller network and are willing to pay less for the good. If the monopolist could commit himself to higher sales, he would be better off, but this commitment is not credible so long as he is the sole producer.'

<sup>23</sup> Let  $A(\theta) \equiv 1 + v(\beta_1 + \theta(\beta - \beta_1)) - (1 - \theta v) \sum_{j=2}^{n+1} q_j$  and write firm 1's inverse demand, (5), as  $p_1 = A(\theta) - (1 - v)q_1$ . Hence,  $\pi_1 = (p_1 - c)q_1 = (A(\theta) - (1 - v)q_1 - c)q_1$ . In any equilibrium where firm 1's output is positive, firm 1's output is given by the first-order condition  $0 = A(\theta) - 2(1 - v)q_1 - c$ , implying  $(1 - v)q_1 = A(\theta) - (1 - v)q_1 - c = p_1 - c$ . Substituting  $p_1 - c = (1 - v)q_1$  into the equation for  $\pi_1$  yields  $\pi_1 = (1 - v)(q_1)^2$ .

outcome—region A in Figure 1—is  $v < \bar{v}(n)$ . In this case, firm 1 prefers autarky if and only if  $q_1^d > q^a$ , where  $q^a$  is given by (8) and  $q_1^d$  by (19). Let  $\underline{m}_1$  denote the installed-base share at which firm 1’s outputs in the two equilibria are equal. Since  $q_1^d$  increases in  $m_1$  but  $q^a$  does not, firm 1 prefers autarky in the interior equilibrium if and only if  $m_1 > \underline{m}_1$ . With the substitution  $\beta_1 = \underline{m}_1\beta$ , solving  $q^a = q_1^d$  for  $\underline{m}_1$  yields

$$(30) \quad \underline{m}_1(c, v, \beta, n) \equiv \frac{(1 - c)n[n(1 - v) - v] + \beta \left[ n^2(1 - v) + 2(1 - v)^2 + n(3 - 6v + 2v^2) \right]}{\beta(n + 2)(1 - v)[n + (n + 1)(1 - v)]}.$$

*Lemma 4 (Profitability of autarky—interior equilibrium).* Whenever network effects are weak enough that the unique equilibrium under autarky is interior, a necessary condition for firm 1 to prefer this outcome over compatibility is that its installed-base share exceeds one half. Formally, for any  $c, v, \beta, n$ , if  $v < \bar{v}(n)$ , then  $\underline{m}_1 > 1/2$ .

The intuition for Lemma 4 was noted by CRT (2000, Proposition 5]: for  $m_1 \leq 1/2$ , autarky yields no quality advantage to firm 1 over the compatible rivals, but reduces all firms’ qualities and hence overall demand by new customers. The curve  $\underline{m}_1$  is shown in Figure 3, which is otherwise identical to Figure 1, except that we confine attention to  $m_1 \geq 1/2$ . The curve  $\underline{m}_1$  partitions region A—in which autarky yields the unique interior equilibrium—into two sub-regions:  $A_1$ , where firm 1’s profit is lower under autarky than under compatibility, and  $A_2$ , where its profit is higher under autarky. It can be shown that the curve  $\underline{m}_1$  passes through the intersection of the curves  $\underline{M}_1$  and  $\bar{M}_1$ .<sup>24</sup>

Now consider tipping equilibria. Where autarky causes tipping from firm 1, firm 1 obviously prefers compatibility. Where autarky causes tipping to firm 1, there is a tradeoff: autarky raises 1’s *share* of new customers (from  $1/(n + 1)$  to 1) but lowers overall demand due to the quality reduction from lost access to the rivals’ installed base. Autarky is more profitable if and

<sup>24</sup> This can be seen as follows. Simple substitution shows that at  $n = 1$  and  $v = 1/2$ , we have  $\underline{m}_1 = \underline{M}_1 = \bar{M}_1 = 1/2$ . Now fix  $c, \beta$ , and  $n$ , with  $n \geq 2$ , and consider  $v'$  slightly less than  $\bar{v}$ . Given  $v'$ , for market shares  $m_1$  satisfying  $\bar{M}_1 < m_1 < \underline{M}_1$  the unique autarky equilibrium is interior. But near the endpoints of this interval, firm 1’s profit approaches that in the two tipping equilibria. At  $(\bar{M}_1, v')$  the autarky equilibrium is tipping from 1, which is clearly less profitable than compatibility; and at  $(\underline{M}_1, v')$  the autarky equilibrium is tipping to 1, which is more profitable than compatibility (see Lemma 5 below). By continuity, there is some market share,  $m_1$ , between  $\bar{M}_1$  and  $\underline{M}_1$  at which firm 1 is indifferent between the two regimes. As  $v'$  increases to  $\bar{v}$ , the interval  $(\bar{M}_1, \underline{M}_1)$  shrinks to a single point, the intersection of the curves  $\bar{M}_1$  and  $\underline{M}_1$ . This intersection also corresponds to  $\underline{m}_1$ .

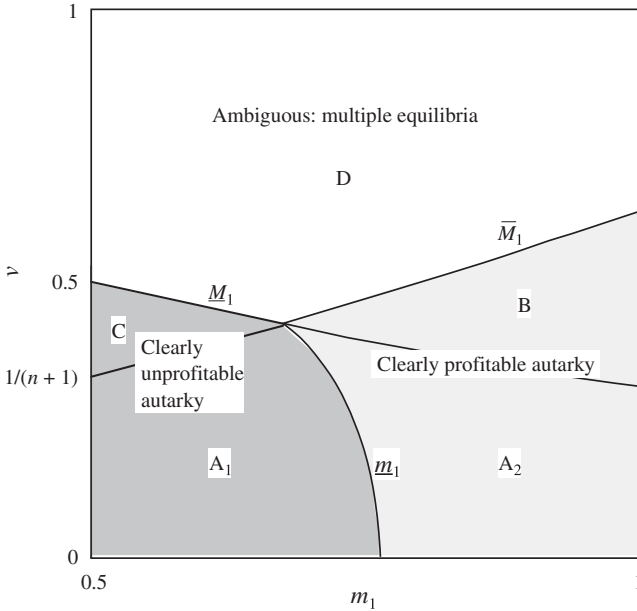


Figure 3  
Profitability of Autarky for Firm 1:  $c = 0.2, \beta = 1, n = 2$

only if  $q_1^{Tip} - q^a > 0$ , where

$$\begin{aligned}
 (31) \quad q_1^{Tip} - q^a &= \frac{(1 - c)n + ((n + 2)\beta_1 - 2\beta)v}{2(n + 2)(1 - v)} \\
 &= \frac{(1 - c)n + ((n + 2)m_1 - 2)\beta v}{2(n + 2)(1 - v)}.
 \end{aligned}$$

The difference in (31) can be negative; hence autarky can be unprofitable even if it yields tipping to firm 1. However, the following sufficient conditions for firm 1 to prefer tipping are obtained by noting that the right hand side of (31) is positive if  $m_1 > 2/(n + 2)$ .

*Lemma 5 (Profitable autarky—tipping to firm 1).* Firm 1’s profit is higher under autarky with tipping to 1 than under full compatibility if (i)  $m_1 > 2/3$  or (ii)  $m_1 > 1/2$  and  $n \geq 2$ .

Summarizing, in Figure 3 autarky is clearly profitable in regions  $A_2$  and B (the latter follows from Lemma 5 because  $n \geq 2$ ); clearly unprofitable in regions  $A_1$  and C; and its profitability in region D is ambiguous.

Note that for  $m_1 \leq 1/2$ , autarky cannot be clearly profitable for firm 1: at the interior equilibrium region (low  $v$ ) this is shown by Lemma 4; in the other

regions with  $m_1 \leq 1/2$ , if tipping to 1 is possible, so too is tipping from 1, but not vice versa (Proposition 1(i)). These remarks yield the following result.

*Proposition 2 (Unprofitable autarky).* If  $m_1 \leq 1/2$ , then autarky (i) is clearly unprofitable to firm 1 for some parameter values and (ii) is never clearly profitable.

Henceforth, we shall focus on the regions where firm 1 finds autarky either clearly profitable or clearly unprofitable. Before proceeding, however, we discuss briefly the multiple equilibria region D. There, if firm 1 were to choose autarky the actual equilibrium would depend on consumers' expectations. Farrell and Klemperer [2004] suggest two possible assumptions about expectations. One possibility is that *expectations track installed bases*; in this case, the outcome under autarky would be tipping to firm 1 given its larger installed base.<sup>25</sup>

By contrast, if *expectations track surplus*, the tipping equilibrium selected is the one that yields greater surplus to new consumers. An argument for this criterion is that if each of the new consumers—who are identical in their network preferences—believes that others will make the same network choice (firm 1 vs. the rivals) as that consumer, then all would 'coordinate' on their preferred equilibrium even absent explicit communication.<sup>26</sup> We establish the following.

*Proposition 3 (Consumers' preference between tipping equilibria).* When both tipping equilibria are possible, consumers prefer tipping from firm 1 if either (i) network effects are not too strong,  $v < 1/2$ , or (ii) firm 1's installed-base share is not too large,  $m_1 \leq 2n/(3n + 1)$ .

In part (ii), an increase in the number of rivals expands the range of firm 1's market share for which consumers prefer tipping from firm 1 since larger  $n$  increases rivals' tipping output and, hence, lowers their quality-adjusted price. For any  $n \geq 3$ , consumers prefer tipping from firm 1 if its share is under 60%. Note that the condition in (ii) is sufficient, not necessary. For example, if  $c = 0$ ,  $\beta = 1$  and  $n = 4$ , it can be shown that consumers prefer tipping from 1 if 1's share is below 80%. Proposition 3 suggests that if consumer expectations track surplus, firm 1 should be quite leery of pursuing autarky in region D unless its market share is rather high.

<sup>25</sup> Besides myopia, ignoring the addition of new consumers and focusing only on installed bases would be rational if new consumers valued compatibility only with installed-base consumers, not with other new ones (Arthur [1989]).

<sup>26</sup> This argument is not dispositive, but it tracks the same logic that predicts that in a simultaneous-move game characterized by multiple Pareto-ranked equilibria, players will choose the dominant equilibrium.

IV(ii). *Comparative Statics and the Underlying Forces*

In light of Proposition 2, to allow for clearly profitable autarky we focus on  $m_1 > 1/2$ . We say that a change in a parameter *encourages autarky* if it weakly expands the scope for clearly profitable autarky, weakly shrinks the scope for clearly unprofitable autarky, and one of these changes is strict ('scope' was defined in Section III.(ii)).

*Installed-Base Share of the Largest Firm* An increase in  $m_1$  makes it more likely that  $m_1 > \underline{m}_1$  (so autarky at the interior equilibrium is more profitable than compatibility), or  $m_1 > \underline{M}_1$  (tipping to 1 is possible), or  $m_1 > \bar{M}_1$  (tipping from 1 is not possible). Such effects can be seen in Figure 3, for example, as a move into  $A_2 \cup B$  or out of  $A_1 \cup C$ . Therefore, we have the following.

*Proposition 4 (Installed-base share).* An increase in  $m_1$  encourages autarky by firm 1.

The intuition is clear: the source of firm 1's potential quality advantage under autarky is its larger number of installed-base customers, an advantage that rises with 1's share.

*Potential for Market Expansion* Increased potential for market expansion *relative* to the installed base arises if the base  $\beta$  is reduced or if firms' marginal cost  $c$  is reduced. Both experiments yield a higher growth rate under compatibility (see Section V). Given  $n \geq 2$ , autarky is clearly profitable for firm 1 if and only if the unique autarky equilibrium is tipping to 1,<sup>27</sup> or is interior and  $m_1 > \underline{m}_1$ ; autarky is clearly unprofitable if the unique autarky equilibrium is tipping from 1, or is interior and  $m_1 < \underline{m}_1$ . Lemma 6 shows how the boundaries of these regions vary with  $c$  and  $\beta$ .

*Lemma 6.*

- (i) If  $v < 1/2$ , then  $\partial \underline{M}_1 / \partial c < 0$  and  $\partial \underline{M}_1 / \partial \beta < 0$ .
- (ii) If  $v > 1/(n+1)$ , then  $\partial \bar{M}_1 / \partial c < 0$  and  $\partial \bar{M}_1 / \partial \beta < 0$ .
- (iii) If  $v < 1/2$ , then  $\partial m_1 / \partial c < 0$  and  $\partial m_1 / \partial \beta < 0$ .

For the ranges of  $v$  given in Lemma 6,  $\underline{M}_1$ ,  $\bar{M}_1$ , and  $m_1$  all exceed  $1/2$ . Geometrically, Lemma 6 implies that a reduction in  $\beta$  or  $c$  pivots the curve  $\underline{M}_1$  counterclockwise through the point  $(m_1, v) = (1/2, 1/2)$  and pivots  $\bar{M}_1$  clockwise through the point  $(m_1, v) = (1/2, 1/(n+1))$ , with the intersection of

<sup>27</sup> When  $n = 1$ , equilibrium tipping to 1 can be unprofitable if  $m_1 < 2/3$ . Rather than impose more complex assumptions in our discussion of the market expansion parameters  $\beta$  and  $c$ , we restrict attention to  $n \geq 2$ .



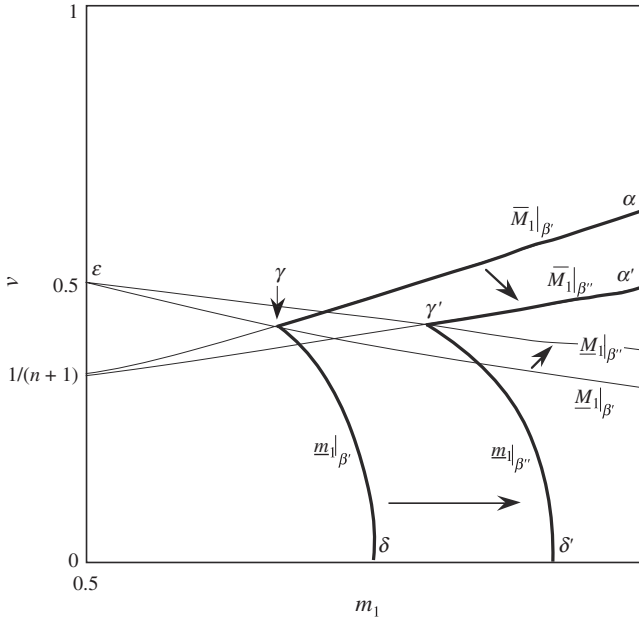


Figure 4  
Effects of Increased Scope for Market Expansion:  $c = 0.2, n = 2, \beta' = 1, \beta'' = 0.5$

these two curves' moving to the right. In addition, the curve  $\underline{m}_1$  shifts to the right. These effects are illustrated in Figure 4, where  $\beta$  is reduced from 1 to 0.5. The initial scenario is depicted by the curves  $\underline{M}_1, \bar{M}_1,$  and  $\underline{m}_1$ . Initially, clearly profitable autarky occurs to the right of the heavily shaded curve  $\alpha\gamma\delta$ ; after  $\beta$  is reduced, the curves  $\underline{M}_1, \bar{M}_1,$  and  $\underline{m}_1$  shift as shown. The region of clearly profitable autarky shrinks to that bounded by the heavily shaded curve  $\alpha'\gamma'\delta'$ ; the region of clearly unprofitable autarky expands from that bounded by  $\delta\gamma\varepsilon$  to that bounded by  $\delta'\gamma'\varepsilon$ . These observations yield the following proposition.

*Proposition 5 (Increased potential for market expansion).* Suppose  $n \geq 2$ . Lower  $c$  or lower  $\beta$  discourages autarky by firm 1: (a) the region of tipping to 1 shrinks, (b) the region of tipping from 1 expands, and (c) the region of the profitable unique interior equilibrium shrinks.

*Tipping regions* Tipping from firm 1 to network  $R$  requires that 1's price net of marginal cost, evaluated at  $R$ 's tipping output and zero output for 1, satisfy  $(1 + m_1\beta - c) - Q_R^{Tip} \leq 0$ ; similarly, tipping from  $R$  to 1 requires  $(1 + m_R\beta - c) - q_1^{Tip} \leq 0$ , where  $m_R \equiv 1 - m_1$ . A change in  $c$  or in  $\beta$  will have two effects on the scope for tipping from network  $j$ : (a) *direct effect*—the

change in the intercept of  $j$ 's inverse demand net of marginal cost,  $(1 + m_j \beta - c)$ , and (b) *indirect effect*—the induced change in the other network's tipping output.

A fall in marginal cost  $c$  discourages tipping from either network ( $j = 1$  or  $R$ ) via the direct effect, but encourages it via the indirect effect. The direct effect is identical for  $j = 1$  or  $R$ . However, the indirect effect is asymmetric: due to intra-network competition among rivals, an equal reduction in marginal cost induces a greater increase in rivals' total tipping output ( $Q_R^{Tip}$ ) than in firm 1's output ( $q_1^{Tip}$ ). As a result, a fall in  $c$  increases the scope for tipping from 1 (the indirect effect dominates the direct) but decreases the scope for tipping to 1 (see the Appendix).

A fall in the installed base  $\beta$  also generates opposing effects, though in reverse directions from a fall in  $c$ : the direct effect of a fall in  $\beta$  makes tipping from either network *more* likely, but the indirect effect makes tipping less likely. Given  $m_1 > 1/2$ , the direct effect is stronger for firm 1 than for rivals—1's inverse-demand intercept falls by  $m_1 v \Delta \beta$  and rivals' by  $(1 - m_1) v \Delta \beta$ —and outweighs the indirect effect *only* in the case of tipping from 1 (see the Appendix). Thus, a lower  $\beta$  increases the scope for tipping from 1 and decreases the scope for tipping to 1.

*Interior equilibrium* Where the unique autarky equilibrium is *interior*, the effects of  $c$ ,  $\beta$ , and  $n$  on the profitability of autarky are better understood using decomposition (9), which expresses the difference in firm 1's inverse demand under autarky versus compatibility:

$$(9) \quad p_1^d|_{Q_R^d} - p_1^a|_{Q_R^a} = \underbrace{(Q_R^a - Q_R^d)}_{\text{Rivals' Contraction (RC)}} (1 - v) - \underbrace{[\beta(1 - m_1) + Q_R^d]}_{\text{Lost Links (LL)}} v.$$

Denote rivals' contraction by  $RC \equiv Q_R^a - Q_R^d$  and firm 1's lost links by  $LL \equiv \beta(1 - m_1) + Q_R^d$ . Autarky becomes relatively less profitable if  $RC$  falls (the benefit of autarky decreases), or if  $LL$  rises (the cost of autarky increases), and is less profitable than compatibility if (9) is negative. Thus, we consider how changing  $c$  or  $\beta$  affects each of these two terms.

Lowering  $c$  increases  $LL$  by raising rivals' autarky output  $Q_R^d$ —a lower marginal cost leads rivals to add more customers in an interior equilibrium, so autarky deprives firm 1 of more links than when marginal cost is higher. Moreover, lowering  $c$  also decreases  $RC$ , the size of rivals' contraction induced by autarky (equivalently, autarky causes a smaller reduction in rivals' interior equilibrium output the lower is marginal cost).<sup>28</sup> Essentially, a cost reduction magnifies the intra-network competition advantage of  $n \geq 2$  rivals over firm 1

<sup>28</sup> It can be verified that  $\partial RC / \partial c = nv[n - 2(1 - v)] / [(n + 2)(1 - v)(2(n + 1)(1 - v)^2 - n)] > 0$ , for all  $n \geq 2$  and  $0 < v < \bar{v}(n)$ .

in expanding their network size, hence quality, putting firm 1 at a quality disadvantage (compared to when  $c$  is higher) *only if it chooses autarky*.<sup>29</sup> Thus, when the common marginal cost is lower, autarky imposes a higher penalty on firm 1 ( $LL$  is larger) and yields a lower benefit ( $RC$  is smaller).

The effect of  $\beta$  on the profitability of autarky at an interior equilibrium is more complicated than the effect of  $c$ . A fall in  $\beta$  may increase or decrease  $LL$ , because of two opposing effects: rivals' installed base  $\beta(1 - m_1)$  falls, but rivals' equilibrium output under autarky  $Q_R^d$  can fall or rise. However, it can be shown that (i) a fall in  $\beta$  will reduce  $RC$  when  $m_1 = \underline{m}_1$ , i.e., at the market share where firm 1's interior-equilibrium profit is equal under autarky and compatibility, and (ii) for all  $v < \bar{v}(n)$ , this effect on  $RC$  outweighs the effect on  $LL$ . On balance, therefore, a lower total installed base reduces the profitability of autarky.<sup>30</sup>

*Number of Rivals* Recall from Proposition 1 that where autarky admits tipping (high  $v$ ), the effect of raising  $n$  is straightforward: rivals' tipping output increases so the region of tipping from 1 expands while tipping to 1 is unchanged; thus, higher  $n$  discourages autarky by firm 1 in tipping regions.

Where autarky yields uniquely the interior equilibrium (low  $v$ ), raising  $n$  lowers firm 1's profit in this equilibrium and under compatibility. Figure 5 shows that the effect of  $n$  on firm 1's preference between autarky and compatibility indeed is ambiguous in this region. Figure 5 fixes  $\beta$  and  $c$  and plots the contour  $\underline{m}_1$  as  $n$  takes the values 1, 2, 4, and 10. For a given  $n$ , the top of the corresponding contour is shown at  $\bar{v}(n)$  because higher values of  $v$  admit tipping equilibria under autarky, so the interior equilibrium is not the unique outcome. The  $m_1$  contours associated with different  $n$  intersect. For high  $v$ , increasing  $n$  can expand region  $A_1$  at the expense of  $A_2$ , discouraging autarky; the reverse occurs in Figure 5 for low  $v$ . Thus, increasing the number of rivals can discourage or encourage autarky by firm 1. Further analysis of the underlying forces using decomposition (9) is provided in the Appendix.

*Strength of Network Effects* The role of  $v$  is ambiguous generally. Where the unique autarky equilibrium is interior, this ambiguity is seen in Figure 5:

<sup>29</sup> The intuition is subtle. The inverse demands facing firm 1 and any rival  $i$ , see (5) and (6), respectively, are parallel. Thus, holding other firms' outputs constant, the desired total expansion of the  $n$  rivals due to a fall in  $c$  exceeds that of firm 1 alone, under compatibility or autarky by firm 1. Under compatibility, this force does not affect firms' relative qualities, since all firms access the same network. Under autarky by firm 1, rivals' larger desired expansion puts firm 1 at a quality disadvantage relative to the original (higher  $c$ ) equilibrium. Consistent with this intuition, (19) and (20) show that under autarky by firm 1, a reduction in  $c$  raises the equilibrium output of any rival  $i$  by more than that of firm 1: for all  $v < \bar{v}(n)$  and  $n \geq 2$ ,

$$\frac{\partial q_i^d}{\partial c} = -\frac{(1 - 2v)}{2(n + 1)(1 - v)^2 - n} < -\frac{(1 - (n + 1)v)}{2(n + 1)(1 - v)^2 - n} = \frac{\partial q_1^d}{\partial c}.$$

<sup>30</sup> It can be verified that, for all  $v < \bar{v}(n)$ ,  $\frac{\partial}{\partial \beta} [(1 - v)RC - v \times LL] \Big|_{m_1 = \underline{m}_1(c, v, \beta, n)} > 0$ .

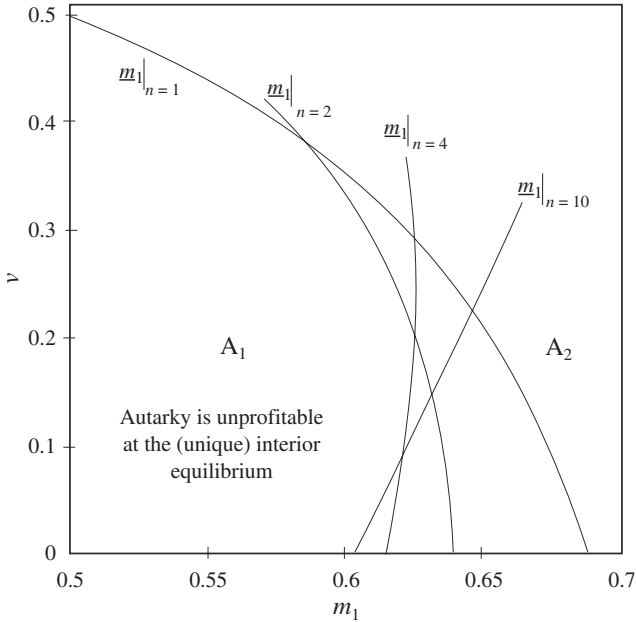


Figure 5

Effects of  $n$  on Profitability of Autarky When the Unique Equilibrium Is Interior:  $c = 0$ ,  $\beta = 5$

$\underline{m}_1$  can be decreasing in  $v$  (for  $n = 1$  or  $2$ ), increasing ( $n = 10$ ), or backward bending ( $n = 4$ ). Thus, raising  $v$  can change the environment from unprofitable to profitable autarky (e.g., for  $n = 2$ ), the reverse (for  $n = 10$ ), or induce two switches (for  $n = 4$ ). Now consider regions (of higher  $v$ ) that admit *tipping* equilibria. Figure 3 shows that raising  $v$  can change the environment to the multiple-equilibria region D starting either from region C of unique tipping from 1—thus encouraging autarky—or from region B of unique tipping to 1—thus *discouraging* autarky.

The ambiguous effect of  $v$  in our model is partially at odds with CRT's statement that, by focusing on low values of  $v$  that ensure an interior equilibrium rather than tipping, they have understated the threat to connectivity.<sup>31</sup> An opposite example is shown in Figure 3, where an increase in  $v$  can move the environment from region  $A_2$ , where firm 1 clearly prefers autarky, to region D where firm 1 may refrain from autarky since the

<sup>31</sup>CRT [p. 455] state that their focus on relatively low  $v$  'stack[s] the deck against the possibility of the extension of market dominance [through incompatibility] . . . Larger network externalities would give rise to "tipping effects" and make it more likely that the industry would be monopolized.'

multiple equilibria including tipping from 1. Thus, tipping possibilities do not systematically push the largest network to incompatibility.

V. APPLYING THE MODEL: MARKET-SHARE SAFE HARBORS

We now show how, with the right information, one can construct market-share safe harbors below which the largest firm is unlikely to prefer autarky if its rivals remain compatible. Such an analysis is relevant, for instance, when reviewing a merger that would expand the largest firm’s customer base and, it is feared, might switch its preference from compatibility to autarky.

Let  $m_1^L$  denote the lowest value of  $m_1$  in the region of ‘clearly profitable autarky’, i.e., the left-most point of  $A_2 \cup B$  in Figure 3. Within the model,  $m_1^L$  is a ‘safe harbor’ in the sense that for any lower market share, firm 1 will *not* find autarky clearly profitable. This threshold would be too permissive if one feared that firm 1 might choose autarky also where multiple tipping equilibria are possible (region D), hoping that the actual outcome would be tipping to it. Subject to this caveat, we show how one can derive  $m_1^L$ .

An obvious challenge is that  $m_1^L$  is a function of the model’s parameters ( $c, v, \beta, n$ ), of which all but  $n$  are difficult to interpret empirically—they have no natural unit of measure. For example,  $c$  is measured relative to the maximum stand-alone value for the product (which we took to be  $\tau = 1$ ); and the installed base  $\beta$  is best understood as measured relative to the number of potential future consumers. Nevertheless, it may be possible to proceed indirectly.

Suppose that for the particular industry, it is estimated—given the current number of firms and assuming the initial *full-compatibility* regime persists—that over the relevant horizon the total number of industry customers would grow by at least some factor  $\Gamma (> 1)$ ; then

$$(32) \quad \Gamma \leq \frac{\beta + (n + 1)q^a}{\beta} = \frac{(n + 1)(1 - c) + \beta(n + 2 - v)}{\beta(n + 2)(1 - v)},$$

where we have substituted for  $q^a$  from (8). Inequality (32) is equivalent to

$$(33) \quad \beta \leq \beta_\Gamma \equiv \frac{(n + 1)(1 - c)}{(n + 2)[(1 - v)\Gamma - 1] + v}.$$

The function  $\beta_\Gamma$  is decreasing in  $\Gamma$ —growth rates exceeding  $\Gamma$  are possible if and only if the initial installed base is smaller than  $\beta_\Gamma$ .<sup>32</sup> Growth by factor  $\Gamma > 1$  is feasible (for an appropriate choice of  $\beta > 0$ ) if and only if the denominator in (33) is strictly positive, a condition that imposes the

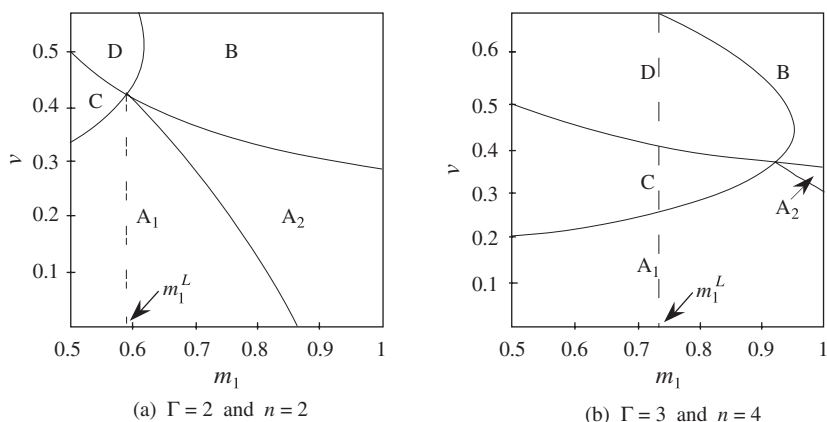
<sup>32</sup> If  $c = 1$ , then the implied growth rate is independent of the size of the installed base. To exclude this possibility, we restricted  $c$  to be less than 1.

following upper bound on the feasible  $v$ :

$$v_{\max} \equiv \frac{(\Gamma - 1)(n + 2)}{\Gamma(n + 2) - 1}.$$

Note that for any  $\Gamma > 1$  and  $n \geq 1$ ,  $v_{\max}$  lies strictly between 0 and 1.

Observe that in (33) the term  $(1 - c)$  is a factor in  $\beta_\Gamma$ . After substitution of  $\beta_\Gamma$  for  $\beta$  and  $m_1\beta_\Gamma$  for  $\beta_1$  in the output formulas for firm 1's output at the interior and tipping equilibria ((8), (19), and (10)),  $c$  enters only through the factor  $(1 - c)$ . Since a firm's equilibrium profit is proportional to the square of its output,  $c$  affects the level of firm 1's profit under autarky, but not whether autarky is more profitable than compatibility. Because we are interested only in the profit ranking, the normalization  $\beta = \beta_\Gamma$  lets us set aside  $c$  and consider only the parameters  $v$  and  $m_1$ , given the number of rivals  $n$  and the expected market growth  $\Gamma$  under compatibility. Figure 6 illustrates how  $m_1^L$  is obtained in two of these cases,  $(n, \Gamma) = (2, 2)$  or  $(4, 3)$ , by displaying the equilibrium regions under autarky for each pair  $(m_1, v)$ . Table I illustrates the properties stated in Proposition 5, that greater scope for market expansion increases the installed-base share needed for the largest firm to profit by choosing autarky.



- Clearly Unprofitable Autarky:** Worse Interior Equilibrium for Firm 1 ( $A_1$ ) or Unique Tipping from Firm 1 (C)
- Clearly Profitable Autarky:** Better Interior Equilibrium for Firm 1 ( $A_2$ ) or Unique Tipping to Firm 1 (B)
- Ambiguous:** Multiple Equilibria (D)

Figure 6

Equilibrium Possibilities and Compatibility Choice: Market Expansion Scenarios

TABLE I  
SAFE HARBORS: THE INSTALLED-BASE SHARE  $m_1^L$  BELOW WHICH AUTARKY IS NOT CLEARLY PROFITABLE FOR FIRM I, GIVEN  $n$  RIVALS AND MARKET GROWTH FACTOR OF  $\Gamma$  UNDER COMPATIBILITY

	$\Gamma$		
	1.5	2.0	3.0
$n = 2$	.544	.595	.710
$n = 4$	.561	.638	.731

## VI. COMPATIBILITY CHOICES OF SMALLER RIVALS

Thus far, we have considered only two regimes—full compatibility and autarky by firm 1—and assumed that firm 1 determines the choice between them. This approach rested on two assumptions: (A1) each firm must offer a uniform compatibility policy to all others and (A2) each smaller rival offers compatibility. One environment in which (A1) and (A2) arise endogenously is the following:

- (a.1) Each firm chooses between a closed standard specific to it and a common open standard.
- (a.2) All  $n + 1$  firms make these choices simultaneously.
- (a.3) The  $n$  smaller rivals have equal-sized installed bases.

Assumption (a.1) dictates (A.1). Given a mild restriction on firms' beliefs, we now show that choosing the open standard is a weakly dominant strategy for each smaller rival, validating (A2).

Given (a.1)–(a.3), consider the choice facing any small rival  $j$ . Incompatibility makes firm  $j$  incompatible with all. If each other firm also has chosen incompatibility, then  $j$ 's choice is irrelevant. Suppose, instead, that a set  $C$  of  $N_c \geq 1$  firms have offered compatibility, and examine the possible Cournot equilibria if  $j$  were to choose incompatibility.

First, suppose the resulting incompatibility equilibrium would be *interior*. If, instead, firm  $j$  adopted compatibility with  $C$ , then it and they become more attractive, so the total output of  $j$  and the  $N_c$  firm(s) increases. Moreover, switching to compatibility causes firm  $j$ 's quality *relative* to any member of  $C$  to either (i) stay the same or (ii) increase.<sup>33</sup> Thus, firm  $j$ 's share of the total output supplied by  $j$  and  $C$  is higher with compatibility than without in case (ii) and equal in (i). As a result, compatibility must increase firm  $j$ 's equilibrium output and hence profit.

<sup>33</sup> Case (i) occurs when  $C$  consists of a single firm that is one of the smaller rivals (since its equilibrium network size is equal to  $j$ 's under compatibility or incompatibility), while (ii) arises in all other cases (either firm 1 is a member of  $C$  or  $C$  consists of two or more firms—in both situations, compatibility expands firm  $j$ 's network reach by more than it expands that of any member of  $C$ ).

Next consider the parameter regions in which incompatibility by firm  $j$  yields *tipping*. If tipping is from firm  $j$ , then firm  $j$  obviously cannot gain from incompatibility. Moreover, by the logic of Proposition 1(i), any parameter values that admit tipping to firm  $j$  as an equilibrium also admit tipping to a network that includes firm 1, whose installed base is larger. Finally, as between tipping to the incompatible firm  $j$  or to a network that includes firm 1, *either* of the consumer expectations considered earlier—expectations track installed bases or track surplus—would select, as the equilibrium, tipping to firm 1's network. Thus, it is natural to assume (a.4):

(a.4) Each smaller rival believes that if it is incompatible, then tipping to it would not occur.

Combining this assumption with the analysis of the interior equilibrium case yields Proposition 6.

*Proposition 6 (Symmetric rivals under uniform compatibility policies).* Given the beliefs (a.4), in the environment described by (a.1)–(a.3), each of the  $n$  symmetric smaller rivals finds compatibility a weakly dominant strategy.

This result validates assumption (A2): symmetric rivals constrained to uniform compatibility policies would each offer compatibility. Hence, the alternative regimes are autarky by firm 1 and full compatibility, and firm 1's preference determines which regime prevails.

Departing from the environment (a.1)–(a.3) can only reduce the scope for full compatibility, by adding ways in which firms might gain from restricting compatibility. Malueg and Schwartz (2006, Appendix 2) illustrate this by considering two departures from (a.1) – (a.3).

*Asymmetric Rivals* When the sizes of rivals' installed bases are sufficiently different (relaxing (a.3)), the following possibility arises. Given incompatibility by firm 1, firm 2 can prefer incompatibility rather than compatibility with the other rival, firm 3, whose installed base is smaller than firm 2's. Moreover, for some parameter values, firm 1 would prefer full compatibility over autarky by firm 1, but would opt for incompatibility if it expects firm 2 to be incompatible with firm 3 (leading to full autarky).

*Targeted Incompatibility by Rivals* Relaxing assumption (a.1), and its counterpart (A1), suppose that each firm can offer compatibility on a firm-by-firm basis. CRT focused on targeting by the largest firm (against one of its two rivals).<sup>34</sup> Complementing that analysis, Malueg and Schwartz [2006]

<sup>34</sup> Malueg and Schwartz [2002] investigated the parameter restrictions implied by CRT's example. We do not attempt here to expand on CRT's analysis of targeted degradation by the largest firm.



focus on targeting by the rivals. They show that even when rivals are symmetric, a majority of them may prefer incompatibility with the rest, and this splintering could shift firm 1's preference away from compatibility. Also, for certain parameter values, rivals collectively can prefer incompatibility with firm 1 even where it prefers the reverse.

## VII. CONCLUSION

A central policy concern in markets characterized by strong network effects is that if one firm attains a high enough share of installed-base customers, it may seek to be incompatible with smaller competitors because its larger customer base then gives it an advantage in competing for new customers. A particularly worrisome scenario is that the market ultimately would then tip to the large firm, yielding monopoly. Moreover, a common intuition holds that when network effects are strong and all links have equal value, the outcome indeed will be tipping to the largest firm if its installed-base share exceeds fifty percent, since its network then offers access to more links than could rivals collectively. We showed that this intuition is flawed when the largest firm faces two or more rivals who are themselves compatible: if the largest firm were to choose incompatibility, the unique equilibrium can be tipping to the rivals. Intra-network competition serves as a commitment to consumers that the rivals' network will expand more aggressively than would a single-firm network, thereby offering a superior network. The rivals' competition-based advantage can outweigh their installed-base disadvantage and support a consistent-expectations equilibrium in which all new consumers select the rivals' network. For a given installed-base share of the large firm, the likelihood of tipping to the rivals increases with their number or, more generally, with the strength of competition among them.

For given installed-base shares, the largest firm is less likely to gain from incompatibility when the potential for market expansion is greater, which arises when the total installed base is smaller or marginal cost is lower. A smaller total installed base weakens the largest firm's advantage from any given *share* of this base; a lower marginal cost strengthens rivals' advantages from intra-network competition. Thus, the risk that the largest firm could profitably refuse compatibility is greater in relatively mature industries such as traditional telephony than in faster growing industries such as the Internet. In rapidly growing markets, an autarky strategy can be unprofitable even if the largest firm controls well over half the installed base.

We conclude with some caveats and extensions to our analysis. First, we mostly assumed that all smaller rivals are compatible, and we presented an environment in which this assumption is valid. Departing from this environment can cause splintering among rivals and induce the largest firm also to prefer incompatibility in some cases where it would remain

compatible if all rivals did. A more complete analysis of mixed compatibility regimes is an area for further work.

Second, we did not allow payments between firms for compatibility, which sometimes may be feasible. Such payments are likely to make compatibility more attractive to the largest firm by offering an instrument for capturing efficiency gains from compatibility, but an open question is the extent to which consumers would share in these efficiency gains.

Third, we assumed that payments by installed-base customers were predetermined and thus unaffected by the firm's compatibility choices, which only influence competition for new customers. This assumption holds in some environments, for example, when installed-base customers purchase a durable good only once; it may also be a reasonable approximation where installed-base customers do purchase again (e.g., upgrade to a new operating system) but relatively far in the future. In other cases, however, some customers may constitute an 'installed base' by not being subject to competition (e.g., because competitors have not built facilities to their region), yet make ongoing payments for their provider's service. Since their willingness to pay will decline if network size is diminished by incompatibility, this effect reduces the largest firm's gain from incompatibility. Forøs, Kind and Sand [2003], however, note an opposing force: when installed-base payments increase with network size, the firm with the largest installed base will also be more aggressive in adding new customers, which reduces the Cournot equilibrium number of customers added by rivals. Incompatibility can then be *less* harmful to the largest firm, because it loses fewer links than when installed-base payments are fixed. A more complete analysis of compatibility incentives when payments from installed-base customers also increase with network size is another fruitful area for future work.

#### APPENDIX

*Proof of Lemma 1.*

It is easily checked that substituting  $v = 1/2$  into (12) yields  $\underline{M}_1|_{v=1/2} = 1/2$ . Also,

$$(A.1) \quad \frac{\partial \underline{M}_1}{\partial v} = \frac{G(v, c)}{(3 - 2v)^2 \beta v^2},$$

where  $G(v, c) \equiv -(1 - c)(3 - 4v) - 2(2 - 2c + \beta)v^2$ . The denominator of (A.1) is strictly positive for  $v \in (0, 1]$ . Moreover,  $G$  is a concave function of  $v$ , achieving at  $v' \equiv (1 - c) / [2(1 - c) + \beta]$  its maximum of

$$G(v', c) = - \frac{(1 - c)(4(1 - c) + 3\beta)}{2(1 - c) + \beta},$$

which is strictly negative for all  $0 \leq c < 1$ . It now follows that  $\partial \underline{M}_1 / \partial v < 0$  for  $0 \leq c < 1$ . Because  $n$  does not enter the expression for  $\underline{M}_1$ , the curve is independent of  $n$ .

*Proof of Lemma 2.*

Substituting  $v = 1/(n + 1)$  into (16) yields  $\bar{M}_1|_{v=1/(n+1)} = 1/2$ , so the  $\bar{M}_1$  curve passes through the point  $(m_1, v) = (1/2, 1/(n + 1))$ . Next, we have

$$(A.2) \quad \frac{\partial \bar{M}_1}{\partial v} = \frac{G(v, c)}{(n + (n + 1)(1 - v))^2 \beta v^2},$$

where

$$G(v, c) \equiv (1 - c)(2n + 1) - 2(1 - c)(n + 1)v + (n + 1)((n + 1)(1 - c) + n\beta)v^2.$$

The function  $G$  is convex in  $v$ , achieving at  $v' \equiv (1 - c)/[(n + 1)(1 - c) + n\beta]$  its minimum of

$$G(v', c) = \frac{(1 - c)n((2n + 1)\beta + 2(1 - c)(n + 1))}{(n + 1)(1 - c) + n\beta},$$

which is strictly positive for all  $c < 1$ . Hence, from (A.2),  $\partial \bar{M}_1 / \partial v > 0$  for all  $0 \leq c < 1$ .

Finally, observe that for all  $0 < v < 1$ ,  $\beta > 0$ , and  $c < 1$

$$\frac{\partial \bar{M}_1}{\partial n} = \frac{(1 - v)(2(1 - c) + \beta v)}{\beta v(n + (n + 1)(1 - v))^2} > 0$$

Thus, in  $(m_1, v)$  space, the graph of  $\bar{M}_1$  shifts rightward as  $n$  increases.

*Proof of Lemma 3.*

It can be shown that

$$(A.3) \quad \underline{M}_1 - \bar{M}_1 = \frac{(2(1 - c) + \beta v)(n + 2 - 4(n + 1)v + 2(n + 1)v^2)}{(3 - 2v)((n + 1)(1 - v) + n)\beta v}.$$

Because  $\beta > 0$ ,  $c < 1$ , and  $0 < v < 1$ , it follows that the difference in (A.3) is zero if and only if  $0 = n + 2 - 4(n + 1)v + 2(n + 1)v^2$ . The two roots of this quadratic equation are

$$1 \pm \sqrt{\frac{n}{2(n + 1)}}.$$

Because we only consider  $v < 1$ , we see (cf. (21))  $\underline{M}_1 = \bar{M}_1$  if and only if

$$v = 1 - \sqrt{\frac{n}{2(n + 1)}} = \bar{v}(n).$$

Because  $\underline{M}_1$  is decreasing in  $v$  and  $\bar{M}_1$  is increasing in  $v$  (Lemmas 1 and 2) and  $\underline{M}_1 = \bar{M}_1$  at  $v = \bar{v}(n)$ , it follows that  $v > \bar{v}(n)$  implies  $\underline{M}_1 < \bar{M}_1$ . Similarly,  $v < \bar{v}(n)$  implies  $\underline{M}_1 > \bar{M}_1$ .

*Proof of Proposition 1.*

- (i) Suppose that the parameters  $(c, v, \beta, n, m_1)$ , with  $m_1 \leq 1/2$ , support an equilibrium with tipping to firm 1. Then  $q_1^{Tip} \leq nq_i^{Tip}$ , where  $q_1^{Tip}$  and  $q_i^{Tip}$  are given by (10) and

(13), respectively. Now

$$\begin{aligned} c &\geq 1 + (1 - m_1)\beta v - q_1^{Tip} \\ &\geq 1 + m_1\beta v - q_1^{Tip} \\ &\geq 1 + m_1\beta v - nq_i^{Tip}, \end{aligned}$$

where the first inequality follows by the assumed tipping to 1, the second because  $m_1 \leq 1/2$ , and the third because  $q_1^{Tip} \leq nq_i^{Tip}$ . It follows that  $c \geq 1 + m_1\beta v - nq_i^{Tip}$ , which implies that tipping from 1 is also an equilibrium. Thus, tipping to 1 cannot be the unique equilibrium if  $m_1 \leq 1/2$ .

- (ii) First suppose  $m_1 > 1/2$  and firm 1 faces a single rival, firm 2. Interchanging the roles of firms 1 and 2, it follows from Proposition 1(i) that tipping from firm 1 (i.e., to firm 2) cannot be the unique equilibrium. Therefore, unique tipping from 1 is possible *only* if  $n \geq 2$ .

Next suppose  $n \geq 2$  and fix  $c < 1$  and  $m_1 > 1/2$ . Define  $v' \equiv [(1/2) + (1/(n+1))]/2$ . Then, for  $\beta > 0$  sufficiently small, (12) and (16) yield  $\underline{M}_1(c, v', \beta) > 1$  and  $\bar{M}_1(c, v', \beta, n) > 1$ , implying equilibrium tipping from 1 is possible but equilibrium tipping to 1 is not. Moreover, in this case there is no interior equilibrium, so tipping from 1 is the unique equilibrium.

- (iii) Consider possible numbers of rivals  $n'$  and  $n''$ , with  $n'' > n'$ . Let  $T'_1$  and  $T'_R$  denote the sets of parameters  $(c, v, \beta, m_1)$  with  $m_1 > 1/2$  and for which  $(c, v, \beta, n', m_1)$  supports equilibrium tipping to 1 and tipping from 1, respectively. Given  $n = n'$ , tipping to 1 is the unique equilibrium for all  $(c, v, \beta, m_1) \in T'_1 \setminus T'_R$ ; <sup>35</sup> tipping from 1 is the unique equilibrium for all  $(c, v, \beta, m_1) \in T'_R \setminus T'_1$ . Define  $T''_1$  and  $T''_R$  analogously for  $n = n''$ . Because equilibrium tipping to firm 1 does not depend on the number of rivals, it follows that  $T'_1 = T''_1$ . Thus, to complete the proof it suffices to show that  $T'_R$  is a strict subset of  $T''_R$ . This follows from Lemma 2, which shows  $\partial \bar{M}_1 > \partial n \geq 0$  for all  $(c, v, \beta, n)$ , with strict inequality for some  $(c, v, \beta, n)$ . Figure 2 depicts the change in scope as  $n$  increases.

*Proof of Lemma 4.*

It suffices to show  $m_1 - 1/2 > 0$  for all  $v < \bar{v}(n)$ . It can be verified that

$$(A.4) \quad m_1 - \frac{1}{2} = \frac{2(1-c)(n(1-v) - v)n + \beta \left[ 2(1-v)^2 + (1-4v+v^2)n + (1-v)v n^2 \right]}{2\beta(2+n)(1-v)[n+(n+1)(1-v)]}.$$

Because  $c < 1$  and  $v < 1/2$  (recall that  $v < \bar{v}(n) \leq 1/2$  is necessary for the equilibrium under autarky to be interior and unique), it is clear that the expression in (A.4) is strictly positive if the coefficient of  $\beta$  in the numerator is strictly positive over the relevant

<sup>35</sup> For any two sets A and B, we define  $A \setminus B$  to be the set of points in A that are not in B.

range. Let the function  $f$  denote this coefficient of  $\beta$ ; that is,

$$f(v, n) \equiv 2(1 - v)^2 + (1 - 4v + v^2)n + (1 - v)vn^2.$$

Observe that

$$(A.5) \quad f(v, 1) = 3 - 7v + 2v^2 = (1 - 2v)(3 - v) > 0$$

for  $v < 1/2$ . Also,

$$(A.6) \quad f(v, 2) = 4 - 8v > 0$$

for  $v < 1/2$ . Next observe that

$$(A.7) \quad \left. \frac{\partial f}{\partial n} \right|_{n=2} = 1 - 3v^2 > 0$$

for all  $v < 1/2$ . Because  $f$  is convex in  $n$  for  $0 < v < 1$  and  $f(v, 2) > 0$  for all  $v < 1/2$ , it now follows from (A.7) that

$$(A.8) \quad f(v, n) \geq f(v, 2) > 0$$

for all  $v < 1/2$  and all  $n \geq 3$ . Inequalities (A.5), (A.6), and (A.8) now establish that the coefficient of  $\beta$  in (A.4) is strictly positive for all  $c < 1$  and  $v < 1/2$ .

*Proof of Proposition 3.*

Suppose that firm 1 pursues autarky and both tipping equilibria are possible. An individual with stand-alone value  $\tau$  obtains net surplus equal to  $\tau + vL - p$  if he buys at price  $p$  from a network with quality  $vL$ , so the preferred tipping equilibrium is the one with the lower ‘quality-adjusted’ price,  $p - vL$ . Tipping to 1 yields each customer surplus  $p_1 - vL_1 = 1 - nq_1^{Tip}$ ; tipping from 1 yields  $p_i - vL_i = 1 - q_i^{Tip}$ ,  $i \neq 1$  (see (2)). Thus, the equilibrium with the lower quality-adjusted price is the one yielding more new customers, a property we exploit next.

The total output under tipping to firm 1 is given by  $q_1^{Tip}$ , in (10). Tipping from firm 1 yields total output equal to  $nq_i^{Tip}$ , where  $q_i^{Tip}$  is given by (13). Consequently, tipping from firm 1 is unanimously preferred by new customers if and only if

$$\begin{aligned} nq_i^{Tip} \geq q_1^{Tip} &\Leftrightarrow \left( \frac{n}{n+1} \right) \left( \frac{1 - c + (\beta - \beta_1)v}{1 - v} \right) \geq \frac{1 - c + \beta_1 v}{2(1 - v)} \\ &\Leftrightarrow c \leq c' \equiv 1 - \left( \frac{(3n + 1)\beta_1 - 2n\beta}{n - 1} \right) v. \end{aligned}$$

Therefore, if  $m_1 \equiv \beta_1/\beta \leq 2n/(3n + 1)$ , then  $c' \geq 1$  for all  $v \geq 0$ . Because we consider only marginal costs  $c < 1$ , it follows that when both tipping equilibria are possible, then consumers prefer the equilibrium with tipping away from firm 1 if  $m_1 \leq 2n/(3n + 1)$ .

Consider next the complementary case of  $m_1 > 2n/(3n + 1) \geq 1/2$ . Because  $m_1 > 1/2$ , a tipping equilibrium from firm 1 exists only if  $v > 1/(n + 1)$  (see Lemma 2) and if  $c$  satisfies (15), which we rewrite as

$$c \leq c'' \equiv \frac{\beta_1(n + 1)v^2 + (n + 1 + \beta n - \beta_1(2n + 1))v - 1}{(n + 1)v - 1}.$$

To conclude that when two tipping equilibria exist consumers will prefer tipping from

firm 1, it suffices to show that  $c' > c''$ . Simple calculation shows

$$(A.9) \quad c' - c'' = \frac{n(n+1)(1-2v)(2\beta_1 - \beta)v}{(n-1)((n+1)v-1)}.$$

The condition  $m_1 > 2n/(3n+1)$  implies  $2\beta_1 - \beta > 0$ ; hence, if  $v < 1/2$ , then (A.9) shows  $c' > c''$ .

*Proof of Lemma 6.*

(i) It is easily calculated that, for  $0 < v < 1/2$  and  $0 \leq c < 1$ ,

$$\frac{\partial \underline{M}_1}{\partial c} = -\frac{1-2v}{(3-2v)\beta v} < 0 \text{ and } \frac{\partial \underline{M}_1}{\partial \beta} = -\frac{(1-2v)(1-c)}{(3-2v)\beta^2 v} < 0.$$

(ii) Also, for all  $v \in (1/(n+1), 1)$  and  $c < 1$ ,

$$\begin{aligned} \frac{\partial \bar{M}_1}{\partial c} &= \frac{1-(n+1)v}{[n+(n+1)(1-v)]\beta v} < 0 \text{ and} \\ \frac{\partial \bar{M}_1}{\partial \beta} &= \frac{(1-c)[1-(n+1)v]}{[n+(n+1)(1-v)]\beta^2 v} < 0. \end{aligned}$$

(iii) Finally, we determine the effects of  $c$  and  $\beta$  on  $\underline{m}_1$ . For all  $v < 1/2$  and  $c < 1$ ,

$$\frac{\partial \underline{m}_1}{\partial c} = -\frac{n[n(1-v)-v]}{\beta(n+2)(1-v)[n+(n+1)(1-v)]} < 0$$

and

$$\frac{\partial \underline{m}_1}{\partial \beta} = -\frac{(1-c)n[n(1-v)-v]}{\beta^2(n+2)(1-v)[n+(n+1)(1-v)]} < 0.$$

*Marginal Cost c and Tipping*

Given  $n \geq 2$ , this differential expansion in tipping outputs (indirect effect of  $c$ ) follows from (10) and (14):

$$(A.10) \quad \left| \frac{\partial q_1^{Tip}}{\partial c} \right| = \frac{1}{2(1-v)} < \frac{n}{(n+1)(1-v)} = \left| \frac{\partial Q_R^{Tip}}{\partial c} \right|.$$

Given  $m_1 \geq 1/2$ , *tipping to 1* is possible for any  $v \geq 1/2$ , so in this range of  $v$  changes in  $c$  do not affect the scope for tipping to 1. For  $v < 1/2$ , (A.10) implies  $|\partial q_1^{Tip}/\partial c| < 1$ . Therefore, a fall in  $c$  reduces the left-hand side of (26)—the condition for tipping to 1—by more than it reduces the right-hand side of (26), making it less likely that condition (26) will be satisfied.

Given  $m_1 > 1/2$ , *tipping from 1* requires  $v > 1/(n+1)$ , so (A.10) then implies  $|\partial Q_R^{Tip}/\partial c| > 1$ . Consequently, a fall in  $c$  reduces the left-hand side of (27)—the condition for tipping from 1—by less than it reduces the right-hand side, making it more likely that condition (27) is satisfied.

*Installed Base  $\beta$  and Tipping*

Given  $m_1 > 1/2$ , *tipping to firm 1* is possible for all  $\beta > 0$  when  $v \geq 1/2$  (Lemma 1), so in this case changes in  $\beta$  do not change the scope for tipping to 1. Next suppose  $v < 1/2$ . To understand the balance of the direct and indirect effects, we rewrite the condition for tipping to 1, (11), as

$$(A.11) \quad 0 \geq (1 - c) \frac{(1 - 2v)}{2(1 - v)} + \left[ (1 - m_1) - \frac{m_1}{2(1 - v)} \right] \beta v.$$

For  $v < 1/2$ , the first term on the right-hand side of (A.11) is positive; therefore, tipping to 1 requires the bracketed term to be strictly negative. Thus, if  $v < 1/2$  and tipping to 1 is possible, then a reduction in  $\beta$  increases the right-hand side of (A.11), thereby decreasing the scope for tipping to 1.

The condition for *tipping from firm 1*, (15), can be rewritten as

$$(A.12) \quad 0 \geq (1 - c) \left( \frac{1 - (n + 1)v}{(n + 1)(1 - v)} \right) + \left[ m_1 - \frac{n(1 - m_1)}{(n + 1)(1 - v)} \right] \beta v.$$

From Lemma 2 it follows that if  $m_1 > 1/2$ , then tipping from 1 requires  $v > 1/(n + 1)$ . It can be shown that if  $m_1 > 1/2$  and  $v > 1/(n + 1)$ , then in (A.12) the coefficient of  $\beta$  is strictly positive. Thus, if  $m_1 > 1/2$  and tipping from 1 is possible, a decrease in  $\beta$  reduces the right-hand side of (A.12), thereby increasing the scope for tipping from 1.

*Number of Rivals and Profitability of Autarky by Firm 1*

For simplicity suppose  $c = 0$ . An increase in  $n$  will increase the lost links to firm 1 if it pursues autarky ( $\partial LL/\partial n > 0$ ). The effect of  $n$  on rivals' contraction,  $RC$ , however, can be positive or negative, depending on the total installed base  $\beta$ . To illustrate this ambiguity, we focus on low  $v$ . It is useful to rewrite (9) as

$$(9') \quad p_1^d|_{Q_R^d} - p_1^a|_{Q_R^a} = v \left[ (1 - v) \frac{RC}{v} - LL \right].$$

It is easily verified that

$$(A.13) \quad \lim_{v \rightarrow 0} \frac{\partial(RC/v)}{\partial n} = \frac{4 + 4\beta(3m_1 - 1) + 2(\beta(3m_1 - 1) - 3)n}{(2 + n)^3}.$$

Equation (A.13) reveals the importance of  $\beta$  in determining whether  $RC$  increases or decreases in  $n$ . If  $\beta$  is small, the expression in (A.13) is sure to be negative, reinforcing the  $LL$  effect; thus, for low  $v$ , higher values of  $n$  reduce the profitability of autarky to firm 1. However, for larger values of  $\beta$  the coefficient of  $n$  in the numerator of (A.13) is positive, implying (A.13) is positive for all  $n$ . Moreover, in this case (large  $\beta$  and low  $v$ ) the positive effect of  $n$  on  $RC$  can outweigh its positive effect on  $LL$ , so an increase in  $n$  can make autarky more profitable to firm 1. Such a case is depicted in Figure 5, which shows that for  $v$  near 0, increases in  $n$  reduce  $\underline{m}_1$ , the minimal market share needed by firm 1 to find autarky profitable. In light of the complicated way that  $n$  interacts with other parameters, we omit a similar analysis of the role of  $n$  at high values of  $v$ .

## REFERENCES

- Armstrong, M., 2004, 'Competition in Two-Sided Markets,' Department of Economics, University College London.
- Arthur, B., 1989, 'Competing Technologies, Increasing Returns, and Lock-In by Historical Events,' *Economic Journal*, 99, pp. 116–131.
- Beard, R.; Kaserman, D. and Mayo, J., 2001, 'Regulation, Vertical Integration and Sabotage,' *Journal of Industrial Economics*, 49, pp. 319–334.
- Bental, B. and Spiegel, M., 1995, 'Network Competition, Product Quality, and Market Coverage in the Presence of Network Externalities,' *Journal of Industrial Economics*, 43, pp. 197–208.
- Besen, S. and Johnson, L., 1986, 'Compatibility Standards, Competition, and Innovation in the Broadcasting Industry,' *Rand Report*, R-3453-NSR.
- Besen, S.; Milgrom, P.; Mitchell, B. and Srinagesh, P., 2001, 'Advances in Routing Technologies and Internet Peering Agreements,' *American Economic Review*, 91, pp. 292–296.
- Brock, G., 1994, *Telecommunications Policy for the Information Age*, (Harvard University Press, Boston).
- Cave, M. and Mason, R., 2001, 'The Economics of the Internet: Infrastructure and Regulation,' *Oxford Review of Economic Policy*, 17, pp. 188–201.
- Church, J. and Gandal, N., 1992, 'Network Effects, Software Provision and Standardization,' *Journal of Industrial Economics*, 40, pp. 85–103.
- Conner, K. and Rumelt, R., 1991, 'Software Piracy: A Strategic Analysis of Protection,' *Management Science*, 37, pp. 125–139.
- Cr mer, J.; Rey, P. and Tirole, J., 2000, 'Connectivity in the Commercial Internet,' *Journal of Industrial Economics*, 48, pp. 433–472.
- David, P., 1985, 'Clio and the Economics of QWERTY,' *American Economic Review*, 75, pp. 332–337.
- Dunham, W., 2004, 'The Determination of Antitrust Liability in *U.S. v. Microsoft*: The Empirical Evidence,' U.S. Department of Justice, Antitrust Division, Economic Analysis Group Discussion Paper No. 04-16.
- Economides, N., 1996a, 'The Economics of Networks,' *International Journal of Industrial Organization*, 14, pp. 673–699.
- Economides, N., 1996b, 'Network Externalities, Complementarities and Invitations to Enter,' *European Journal of Political Economy*, 12, pp. 211–233.
- Economides, N. and Flyer, F., 1997, 'Compatibility and Market Structure for Network Goods,' NYU Stern School of Business Discussion Paper No. 98–02.
- Ennis, S., 2002, 'Network Connection and Disconnection,' U.S. Department of Justice, Antitrust Division, Economic Analysis Group Discussion Paper No. 02–5.
- European Commission, *Commission Decision of 8 July 1998 declaring a concentration incompatible with the common market and the EEA Agreement* (Case No. IV/M.1069 – WorldCom/MCI).
- European Commission, *Commission Decision of 28 June 2000 declaring a concentration incompatible with the common market and the EEA Agreement* (Case No. COMP/M.1741 – MCI WorldCom/Sprint).
- Farrell, J. and Gallini, N., 1988, 'Second Sourcing as a Commitment: Monopoly Incentives to Attract Competition,' *Quarterly Journal of Economics*, 108, pp. 673–694.
- Farrell, J. and Klemperer, P., 2004, 'Coordination and Lock-In: Competition with Switching Costs and Network Effects,' [www.pauklemperer.org](http://www.pauklemperer.org).
- Farrell, J. and Saloner, G., 1986, 'Installed Base and Compatibility: Innovation, Product Preannouncements and Predation,' *American Economic Review*, 76, pp. 940–955.



- Faulhaber, G., 2002, 'Network Effects and Merger Analysis: Instant Messaging and the AOL-Time Warner Case,' *Telecommunications Policy*, 26, pp. 311–333.
- Forø, O., Kind, Hans J. and Sand, J., 2003, 'Do Incumbents Have Incentives to Degrade Interconnection Quality in the Internet?' NCFS Working Paper Series in Economics and Management, No. 10-03, University of Tromsø, December.
- Gandal, N., 2002, 'Compatibility, Standardization and Network Effects: Some Policy Implication,' *Oxford Review of Economic Policy*, 18, pp. 80–91.
- Gilbert, R. and Katz, M., 2001, 'An Economist's Guide to *U.S. v. Microsoft*,' *Journal of Economic Perspectives*, 15, pp. 25–44.
- Katz, M. and Shapiro, C., 1985, 'Network Externalities, Competition, and Compatibility,' *American Economic Review*, 75, pp. 424–440.
- Katz, M. and Shapiro, C., 1986, 'Technology Adoption in the Presence of Network Externalities,' *Journal of Political Economy*, 94, pp. 822–841.
- Klein, B., 2001, 'The Microsoft Case: What Can a Dominant Firm Do to Defend Its Market Position?,' *Journal of Economic Perspectives*, 15, pp. 45–62.
- Koski, H. and Kretschmer, T., 2004, 'Survey on Competing in Network Industries: Firm Strategies, Market Outcomes and Policy Implications,' *Journal of Industry, Competition and Trade*, 4, pp. 5–31.
- Malueg, D. and Schwartz, M., 2002, 'Interconnection Incentives of a Large Network,' Georgetown University Department of Economics Working Paper 01-05, August 2001, revised January 2002.
- Malueg, D. and Schwartz, M., 2006, 'Compatibility Incentives of a Large Network Facing Multiple Rivals,' Georgetown University Department of Economics Working Paper 2006.
- Matutes, C. and Régibeau, P., 1996, 'A Selective Survey of the Economics of Standardization: Entry Deterrence, Technological Progress and International Competition,' *European Journal of Political Economy*, 12, pp. 183–209.
- Rochet, J.-C. and Tirole, J., 2004, 'Two-Sided Markets: An Overview,' mimeo, IDEI, Toulouse.
- Rohlfis, J., 1974, 'A Theory of Interdependent Demand for a Communications Service,' *Bell Journal of Economics*, 5, pp. 16–37.
- Salop, S. and Scheffman, D., 1987, 'Cost-Raising Strategies,' *Journal of Industrial Economics*, 36, pp. 19–34.
- Schwartz, M. and Thompson, E., 1986, 'Divisionalization and Entry Deterrence,' *Quarterly Journal of Economics*, 101, pp. 307–321.
- Takeyama, L., 1994, 'The Welfare Implications of Unauthorized Reproduction of Intellectual Property in the Presence of Demand Network Externalities,' *Journal of Industrial Economics*, 42, pp. 155–166.
- Telecommunications Act of 1996*. Pub. L. 104-104, Stat. 56 (codified at 47 U.S.C. §151 *et seq.*).
- U.S. Department of Justice, Complaint of the U.S. Department of Justice, *United States of America v. WorldCom, Inc., and Sprint Corporation*, Civil Action No. CV 01526, United States District Court for the District of Columbia, filed 27 June, 2000.
- WorldCom, Inc. and Sprint, Response of WorldCom, Inc. and Sprint Corporation to The European Commission's Statement of Objections dated 3 May, 2000 (Case No. COMP/M.1741 – MCI WorldCom/Sprint), Response submitted 22 May, 2000.